## The Probable Beginning

The development starts with an analysis of the beginning of the universe. The details of what really happened when the Origin occurred cannot be definitely known, but analysis can reveal the more probable possibilities. These can then be matched to development required to yield a logical total development from the Origin to the verified descriptions of which 20th Century physics and cosmology consist. The resulting total development can then be tested in terms of descriptions and predictions that it makes and implications and experimental tests inherent in it.

The starting point is the assumption that, when the primal nothing changed into something and anti-something as a probabilistically inevitable interruption of what would otherwise have been an infinite duration of the primal nothing, the simplest or minimum conservation-maintaining interruption that could occur is what occurred. There are two reasons for this assumption. Occam's Razor, which we have already encountered, calls for the simplest hypothesis as the most likely. More importantly, or perhaps the same thing, if an essentially spontaneous event is to occur for the sole purpose of interrupting an otherwise total nothing, then very little interrupting event is needed; the barest minimum of something is sufficient to interrupt, to be a change in absolute nothing. There is no call, no reason for anything more. So, while the interruption could have been otherwise, it was probably as simple and minimum as possible.

The immediate resulting implications of this are that the "something" (and its "anti-something") were each:

- one thing, not a number of things,
- one continuous entity, not a mass of "particles"
nor anything having parts,
- one kind of thing, not two, several, etc,
- of simple form,
- as uniform as possible throughout,
- of minimum "tangibility" or "substantiality".

It can further be concluded from all of the foregoing that the change from nothing to something took place in the simplest possible fashion and that the change itself and its consequences were / are subject to the restriction of the impossibility of infinity as well as the requirement of conservation. Resulting further implications are:

- the rate of change was finite; that is, rather than an instantaneous jump from nothing to something there was a gradual transition at a finite rate of change;
- the rate of change of the rate of change (in calculus the 2 nd derivative) was also finite; that is, rather than an instantaneous jump from zero rate of change there was a gradual transition;
- similarly for the rate of change of the rate of change of the rate of change (in calculus the 3rd derivative) and so on ad infinitum.
(See detail notes DN 1 - Differential Calculus, Derivatives, immediately following this section, for an explanation of rate of change and derivatives.)

This last set of points requires that the change took place in a manner describable either as a natural exponential or some form of sinusoid. (This contention is proven at detail notes DN 2 - Analysis: All Derivatives Finite, Selecting $U(t)$, immediately following $D N$ 1.) See Figure 10-1, below. In the figure, the equations which follow, and the discussion of them on the next page, $t_{0}$ is the instant of the beginning and $U(t)$ is the quantity (Universe) that changes with time, $t$.




Figure 10-1
The curves of the above figure, stated mathematically, are that $U(t)$ is of the form:

| (10-1) | $\begin{aligned} & U(t)=\left(\varepsilon^{t-t_{0}}-1\right) \\ & U(t)=0 \end{aligned}$ | $\begin{aligned} & t \geq t_{0} \\ & t<t_{0} \end{aligned}$ |
| :---: | :---: | :---: |
| (10-2) | $\begin{aligned} & U(t)=[1-\operatorname{Cos}(2 \pi f t)] \\ & U(t)=0 \end{aligned}$ | $\begin{aligned} & t \geq t_{0} \\ & t<t_{0} \end{aligned}$ |
| (10-3) | $\begin{aligned} & U(t)=[1-\operatorname{Cos}(2 \pi f t)] \\ & U(t)=\text { any non } \infty \text { form } \\ & U(t)=0 \end{aligned}$ | $\begin{aligned} & t \leq 1 / 2 f \text { and } t \geq t_{0} \\ & t>1 / 2 f \\ & t<t_{0} \end{aligned}$ |

The graph of equation 10-3 is the case where $U(t)=2$ for $t \geq^{1} / 2 f$ which is the simplest form of that case. In each case the graph depicts the "something" as $U(t)$ in $+U$ and the "anti-something" as $-U(t)$ in $-U$.

The change might have been a change in kind, a gradual change from nothing to "mature" something, or a change in size, a gradual growth from nothing to some size, or both. The change could have been for a brief time or it could still be going on now. But, all of these comments raise the question of why there should be growth at all? As soon as any change occurred it was enough to interrupt eternity even though it were an infinitesimal change. In fact, size or period of time are of no meaning here because there is nothing to compare to or measure by. Whatever amount of change occurred is what occurred. Whatever time it took, or went on for, is what happened. Twice as much or half as much have no meaning and make no difference to the matter of simplicity. The important thing is that something was more (or other) than nothing.

Then what about infinity $(\infty)$; is the foregoing form equation $10-1$ possible ? No, it involves unbounded infinities. $U(t)$ and $-U(t)$ are each singly open-ended, being two variables: $U(t)$ with scale range from 0 to $+\infty$ and $-U(t)$ with scale range from 0 to $-\infty$. By analogy an elephant cannot be infinitely large. The fact that it has zero at one end of its size scale range does not relieve the need for an upper limit on size.

Time, on the other hand, is a doubly open-ended variable. Its scale range is from $-\infty$ to $+\infty$. An open end to the past had already occurred, so nothing could be done about that. It was interruption of the open end to the future that was necessary, that was sufficient to make the duration of the primal nothing less than infinite, and that led to our universe.

- An alternative to this theorem on infinity and scale range relative to time is to define time such that it did not exist prior to $t_{0}$. In the metric or topological sense this is valid and is a conclusion of the general theory of relativity, but as time has been defined in Part II for the purpose here, namely that time is the potentiality for duration, it is not valid to define it as not existing prior to $t_{0}$.
- The only other alternative to accepting the theorem's contention that the one interruption was sufficient to prevent infinite duration even though time already extended infinitely into the past would be an infinite chain of nothing -- interrupted by something -- then a reversion back to nothing (or to something else) -another interruption -- and so on, which is absurd.

Concerning the other two alternative forms for $U(t)$, equations 10-2 and 10-3, there is less basis for choice in terms of the discussion so far. It can be argued that equation $10-2$ is more simple than equation $10-3$. Equation $10-3$ requires two changes, the one at $t_{0}$ and a second one changing from the $[1-\operatorname{Cos}(2 \pi f t)]$ form to something else at $t=1 / 2 f$ furthermore, oscillations, waves, are ubiquitous in our universe from oceans, violin strings and pendulums to sound, light and electron orbits. Thus the preferred and, therefore, selected
form is that of equation 10-2. Subsequent development of details of the mechanics of the universe will reinforce this choice.
(Detail notes DN 2-Analysis: All Derivatives Finite, prove that $U(t)$ must be of the form $[1-\operatorname{Cos}(2 \pi f t)]$.)

This choice leads to two further conclusions:

- the frequency, $f$, of the sinusoidal oscillation is very large, and
- the nature of the change is one of amount or "degree" or "maturity" of the something.

The frequency would have to be either very large or very small -- high enough so that it is not detected or noticed by us in every day life or so low that it appears to us as no change at all in our experience. But, the latter choice completely eliminates the oscillation playing any significant part in the behavior of the universe as we detect it. Since the ubiquitousness of waves was a major reason for selecting the oscillatory option, the oscillation must play a part in nature as we know it. Therefore, the high frequency is selected.

The change can hardly be one of gross size if it is going on right now at high frequency as has just been concluded. One can conceive of the fundamental "substance", the "something" of the universe flashing into and out of existence in an oscillatory fashion at a rate so high that we neither detect nor notice it at all. But, it is difficult to entertain a concept of reality flashing from zero to full size in such a fashion. (Actually, the reality that we know is not "flashing into and out of existence ...." As will be seen, our reality is more the oscillation itself than what is oscillating, and a continuing oscillation is our steady, constant reality.)
(These conclusions about frequency and gross size variation, while valid, are subject to some revision as will be seen when further details of the Beginning are developed in a later section of this Part III.)

Thus the hypothesis is that the interruption that gave us our universe was the starting of an oscillation, present to us at a very high frequency and of the general $[1-\operatorname{Cos}(2 \pi f t)]$ form of the density, as the variation will be hereafter referred to, of the medium, as what is oscillating will be hereafter referred to. But, before proceeding further with the discussion two other questions must be addressed:

- the conservation-maintaining "anti-something" and - the medium.

All of the discussion so far must apply equally to the "anti-something", the region of negative $U$ in the graphs. This is because the exact same reasoning as for $+U$ applies to $-U$ and, after all, they are not distinguishable in the discussion. The terms " + " and " - " are merely terms of convenience for two equal and opposite unknown things. We probably tend to think of our universe as the " + ", but that is meaningless and irrelevant. There can be no objective designation of $+U$ and $-U$, no way to identify one versus the other. The question arises, however, as to whether $+U$ and $-U$ are co-located or separate. The answer is that they must be co-located. Their function relative to each other
is to maintain overall conservation from the beginning. They initially are identical except for the $+/-$ oppositeness and therefore each must obey the same laws thereafter. Those laws practice conservation, consequently conservation will be maintained if the beginning conserved, which it did.

Since $+U$ and $-U$ are co-located it opens the possibility that the universe that we know and exist in could be the combined or integrated result of both $+U$ and $-U$. This turns out to be the case as will be developed in subsequent sections. (It is interesting to observe here that our universe being the integrated result of an initial beginning and its opposite relates to (presumably is the underlying cause of) the dialectical nature of reality, the ying and yang of oriental philosophy.)

The question of what the medium is can only be answered in terms of its characteristics, what it does and how. It will be fruitless to attempt to use human terms (gas, jelly or whatever) to describe that which so far underlies the things our vocabulary was developed to describe. We might as well call it a chocolate sauce or a raspberry mousse. The characteristics of the medium, which are its definition, must await developments in the following sections. For now the only possible observation is the set of criteria for simplicity set out early in this section. The medium is:

```
- one thing, not a number of things,
- one continuous entity, not a mass of "particles"
    nor anything having parts,
- one kind of thing, not two, several, etc,
- of simple form,
- uniform throughout (except, of course, for the
    density variations, the oscillation),
- of minimum "tangibility" or "substantiality".
```


## DETAIL NOTES 1

## Differential Calculus, Derivatives

A quantity that can have various different values is called a "variable". When a variable, for example $U$, has its value determined by the value of another variable, for example $t$, the $U$ is said to be "a function of" $t$, which is written as $U(t) . U(t)$ is called the "function" and $t$ is called the "argument". Because the value of $U$ depends on the value of $t, U$ is called the "dependent variable" and $t$ is called the "independent variable". If the relationship between $U(t)$ and $t$ is known, and if $U(t)$ has one and only one value for each value of $t$ (it is "single valued") and if $U(t)$ is "smooth" and "continuous" (meaning that its graphical depiction has no discontinuities and no sharp corners); then the rate of change of $U(t)$ with respect to a change in $t$ can be found as set out in the following.

For any value of the argument, $t$, the function has the value $U(t)$. If the argument then increases by $\Delta t$ ( $\Delta$, read as "delta", means "change in"), to the new value $(t+\Delta t)$, the function then changes to the new value $U(t+\Delta t)$. The changes are, then

```
(DN1-1) change in U(t) = U(t + \Deltat) - U(t)
(DN1-2) change in t = (t + \Deltat) - t = \Deltat
(DN1-3) change in U(t) U(t + \Deltat) - U(t)
    \Deltat
```

Of course, this rate of change is a kind of net average rate over the interval of the change, $\Delta t$. The actual rate at any point in that interval could be quite different. Figure DN1-1, below, is a graphical illustration of the situation.


Figure DNI-I

Clearly the rate of change obtained so far is actually the rate of change (or slope) of the straight line, $L$, of the above figure and only a crude approximation to the rate of change of $U(t)$ in the interval. But, if $\Delta t$ should become increasingly smaller so that the upper right point (circled) of intersection of the straight line, $L$, and the curve of $U(t)$ moves left and down (always staying on the curve of $U(t)$, however) increasingly closer to the lower left such point (circled), then the rate of change of the line, $L$, becomes an increasingly closer approximation to the actual rate of change of $U(t)$ for the value $t$. If we let $\Delta t$ become infinitesimal then the rate of change of line $L$ is essentially identical to that of $U(t)$ at $t$.

$$
\text { (DN1-4) } \begin{aligned}
& \text { Rate of change } \\
& \text { of } U(t) \text { with } \\
& \text { respect to } t
\end{aligned}=\begin{gathered}
{[\text { Limit }} \\
\Delta t \rightarrow 0]
\end{gathered} ~ o f ~\left[\begin{array}{c}
U(t+\Delta t)-U(t) \\
\Delta t
\end{array}\right]
$$

This quantity is called the "derivative" of $U(t)$ and is its rate of change.
The symbol for this quantity, which is read as "d $u$ of $t, d t$ ", is

$$
\begin{aligned}
\frac{d U(t)}{d t} & \equiv \frac{\text { infinitesimal change in } U(t)}{\text { infinitesimal change in } t} \\
& \equiv \frac{\text { "differential } U(t) "}{\text { "differential } t "}
\end{aligned}
$$

One can also take the derivative (the rate of change) of a derivative, and so on ad infinitum. These are called the second, third, ... derivatives and are symbolized as:
(DN1-5)

$$
\frac{d^{2} U(t)}{d t^{2}}, \quad \frac{d^{3} U(t)}{d t^{3}}, \quad \cdots
$$

The following finds the first derivative of a quantity raised to a constant power and multiplied by some other constant, that is letting $U(t)=C t^{P}$, where $C$ is the constant multiplier and $P$ is the power.

$$
\begin{aligned}
& \frac{\Delta U(t)}{\Delta t}=\frac{U(t+\Delta t)-U(\Delta t)}{\Delta t}=\frac{C \cdot(t+\Delta t)^{P}-C \cdot t^{P}}{\Delta t} \\
& \begin{array}{l}
\text { Using the binomial expansion theorem of } \\
\text { algebra the } C \cdot(t+\Delta t) P \text { expanded gives }
\end{array} \\
= & \frac{C \cdot\left[t^{P}+\frac{P \cdot t^{P-1} \cdot \Delta t}{1}+\frac{P \cdot(P-1) \cdot t^{P-2} \cdot \Delta t^{2}}{1 \cdot 2}+\cdots\right]-C \cdot t^{P}}{\Delta t} \\
= & C \cdot P \cdot t^{(P-1)+C \cdot P \cdot(P-1) \cdot t(P-2) \cdot \frac{\Delta t}{2}+\cdots}
\end{aligned}
$$

THE ORIGIN AND ITS MEANING

$$
\begin{aligned}
(D N 1-7) & \frac{d U(t)}{d t}=\begin{array}{c}
{[\text { Limit }} \\
a s
\end{array} \text { of }\left[\frac{\Delta U(t)}{\Delta t}\right] \\
\begin{aligned}
d \mathrm{dU}(\mathrm{t}) \\
d t
\end{aligned} & =\begin{array}{c}
{[\text { Limit }} \\
\Delta \mathrm{as} \rightarrow 0]
\end{array}\left[\mathrm{C} \cdot \mathrm{P} \cdot \mathrm{t}^{\mathrm{P}-1}+\mathrm{C} \cdot \mathrm{P} \cdot(\mathrm{P}-1) \cdot \mathrm{t}^{\mathrm{P}-2} \cdot \frac{\Delta \mathrm{t}}{2}+\ldots\right] \\
& =\mathrm{C} \cdot \mathrm{P} \cdot \mathrm{t}^{\mathrm{P}-1}
\end{aligned}
$$

Thus it is seen that
(DN1-8)

$$
\frac{d[C \cdot U(t)]}{d t}=C \cdot \frac{d U(t)}{d t} \quad \text { and } \quad \frac{d\left[t^{P}\right]}{d t}=P \cdot t^{P-1}
$$

By similar methods it can be shown that each of the following derivative relationships are valid (where $u$ and $v$ are functions of $t: u=u(t)$ and $v=v(t)$ and $C$ is a constant multiplier and $P$ is a constant exponent.
$(D N 1-9)$
$(D N 1-10)$

$\frac{d(u \pm v)}{d t}=\frac{d u}{d t} \pm \frac{d v}{d t}$
(DN1-11) $\frac{d(u \cdot v)}{d t}=u \cdot \frac{d v}{d t}+v \cdot \frac{d u}{d t}$
(DN1-12) $\quad \frac{d\left({ }^{u} / v\right)}{d t}=\frac{u \cdot \frac{d v}{d t}-v \cdot \frac{d u}{d t}}{v^{2}}$
(DN1-13) $\frac{d\left(u^{P}\right)}{d t}=P \cdot u^{P-1} \cdot \frac{d u}{d t}$
(DN1-14)

$$
\text { If } \frac{d u}{d t}=v, \text { then } d u=v \cdot d t
$$

## DETAIL NOTES 2

## Analysis: All Derivatives Finite, Selecting $U(t)$

If not familiar with differential calculus see detail notes $D N 1$ Differential Calculus, Derivatives before proceeding here.

While differential calculus depends on the function being smooth and continuous the case of non-continuous functions can be addressed.


Figure DN2-1

Consider the function

$$
\begin{array}{lll}
\text { (DN2-1) } & U(t)=0 & t \leq 0 \\
& U(t)=t^{2} & t>0
\end{array}
$$

which might be a theoretical candidate for $U(t)$ at the Beginning of the universe and is graphically depicted at the left.

The first derivative is

$$
\begin{array}{ll}
(D N 2-2) & \frac{d U(t)}{d t}=0
\end{array} t<0
$$

and is unstated for $t=0$ because $d U(t) / d t$ is not smooth there (even though it "looks" smooth). It is graphically depicted to the left.

Now consider the second derivative, also depicted to the left.

$$
\begin{array}{rlr}
(\text { DN2-3) } & \frac{d^{2} U(t)}{d t^{2}}=0 & t<0 \\
& \frac{d^{2} U(t)}{d t^{2}}=2 & t>0
\end{array}
$$

Note that it is clearly discontinuous at $t=0$ where it instantaneously jumps from 0 to 2 .

The third derivative, which is the rate of change of the second derivative must be infinite at $t=0$ to produce the instantaneous jump from 0 to 2 . Clearly, this cannot happen in the real world. It is such a condition which is unacceptable in a candidate function for $U(t)$ at the Beginning of the universe.

The only way to avoid this condition of an infinite derivative somewhere is to have a function with an endless family of finite, non-zero derivatives; that is, some derivatives may be zero at $t=0$ but there must always be further non-zero higher derivatives, which requires that the functional form of every derivative must be non-zero.

One can conceive theoretically of the idea of a function for which all derivatives are non-zero and no two are alike (in a general sense analogous to the pattern of digits in an irrational number), but it is not likely that such a function can exist. In any case the more certain and more simple way to achieve all nonzero derivatives is a repeating derivative function, the two simplest examples of which are:

and are analyzed as follows.

## Case (a), Functions Satisfying Equation DN2-4

The function meeting this requirement is

$$
\begin{aligned}
(D N 2-6) \quad \varepsilon^{t} & =1+t+\frac{t^{2}}{2 \cdot 1}+\frac{t^{3}}{3 \cdot 2 \cdot 1}+\cdots \\
& =1+t+\frac{t^{2}}{2!}+\frac{t^{3}}{3!}+\cdots
\end{aligned}
$$

where the symbol "!" is read "factorial" and the meaning is as in the above equation. Taking the first derivative term by term as
(DN2-7)

$$
\begin{aligned}
\frac{d \varepsilon^{t}}{d t} & =0+1+\frac{2 t}{2}+\frac{3 t^{2}}{3 \cdot 2}+\cdots \\
& =1+t+\frac{t^{2}}{2}+\cdots \quad=\varepsilon^{t}
\end{aligned}
$$

and the original function results as is required by DN2-4.
This is the prime case of a function that satisfies the requirement of all derivatives existing in functional form. In general these are:

$$
(D N 2-8) \quad U(t)= \pm \varepsilon^{ \pm t} \quad \text { or } \quad U(t)= \pm A \varepsilon^{ \pm t}
$$

for which all derivatives are

$$
\text { (DN2-9) } \quad \frac{d^{n_{U}}(t)}{d t^{n}}= \pm \varepsilon^{ \pm t} \quad \text { or } \quad \frac{d^{n} U(t)}{d t^{n}}= \pm A \varepsilon^{ \pm t}
$$

(The number, $\varepsilon$, a constant, is called the "natural logarithmic base" since if $u=\varepsilon^{t}$ then $t=\log _{\varepsilon} u$. The value of $\varepsilon$ is calculated by setting $t=1$ in equation DN2-6 and calculating, the result being 2.71828....)

The function $\varepsilon^{t}$ is not suitable for $U(t)$ at the Beginning of the universe, however, because its value at $t=0$ is not zero. In fact it is zero only at $t=-\infty$. A function that might seem usable, however, would be

$$
\begin{array}{rlrl}
(D N 2-10) & & t \leq 0 \\
U(t) & =0 & t>0 \\
U(t) & =\varepsilon^{t}-1 & t^{2} \\
& =t+\frac{t^{3}}{2!}+\frac{t}{3!}+\cdots
\end{array}
$$

which does have zero value at $t=0$ and otherwise meets the derivatives requirement sufficiently.

## Cases (b to e), Functions Satisfying Equation DN2-5

Turning to functions that meet the requirement that the second derivative equal the original function per equation DN2-5 there are four such functions.
(DN2-11)

$$
\begin{array}{ll}
(D N 2-11) & \text { Case (B): } \\
(D N 2-12) & C t)=1+\frac{t^{2}}{2!}+\frac{t^{4}}{4!}+\cdots \\
(D N 2-13) & \text { Case (C): } U(t)=1-\frac{t^{2}}{2!}+\frac{t^{4}}{4!}-\cdots \\
(D N 2-14) & \text { Case (D): } \quad U(t)=t+\frac{t^{3}}{3!}+\frac{t^{5}}{5!}+\cdots
\end{array}
$$

(DN2-12)
(DN2-14)

Cases (в) and (с) have the same problem that $\varepsilon^{t}$, case (A), had, that the value of $U(t)$ is not zero at $t=0$. Just as with case (A), they would appear to become satisfactory if a constant 1 is subtracted from each of them.

These five candidate functions can be described and summarized as their exponential equivalents as in Figure DN2-2, below. The relationships in the table can be verified by substitution using the formula for $\varepsilon^{t}$ as given in equation DN2-6. The $i$ quantity is defined as a constant, the "imaginary coefficient", the square root of -1 , that is:

$$
i=\sqrt{-1} .
$$

| Case | Function | Name of Function | Candidate U(t) |
| :---: | :---: | :---: | :---: |
| (A) | $\varepsilon^{t}$ | Natural exponential | $\pm\left[\varepsilon^{t}-1\right]$ |
|  | $\varepsilon^{t}+\varepsilon^{-t}$ |  |  |
| (B) | $2$ | Hyperbolic cosine | $\pm[\operatorname{Cosh}(t)-1]$ |
|  | $\varepsilon^{i t} \varepsilon^{-i t}$ |  |  |
| (c) |  | Cosine | $\pm[\operatorname{Cos}(t)-1]$ |
|  | $\varepsilon^{t}-\varepsilon^{-t}$ |  |  |
|  | 2 |  |  |
|  | $\varepsilon^{i t} \varepsilon^{-i t}$ |  |  |
| (E) |  | Sine | $\pm \operatorname{Sin}(\mathrm{t})$ |
|  | $2 i$ |  |  |

Figure DN2-2
These candidates all satisfactorily meet the requirement for a continuous family of derivatives so that the kind of unacceptable problem as encountered in the example of $U(t)=t^{2}$ at the opening of these detail notes is avoided, that is all derivatives are finite. But, there are other requirements that the successful candidate function must meet.

## Using the Remaining Criteria to Select U(T)

Two other criteria must be met by the successful candidate function or functions:

- the function must not be open-ended, that is it cannot ever have an infinite amplitude of $U(t)$, and
- the function must smoothly match the $U(t)=0$ condition at $t=0$.

The first criterion eliminates cases (A), (B) and (D) each of which goes to an infinite value of $U(t)$ one or more times. To satisfy the second criterion the tangent to $U(t)$ at $t=0$ must be identical to the tangent to the function for $t \leq 0$, which is the horizontal $t$-axis. The condition is satisfied if the first derivative of $U(t)$ equals zero at $t=0$. Only cases (B) and (C) meet this requirement. Figure DN2-3, below, summarizes these results.

| Case | Candidate U(t) | Open-Ended <br> Goes to Infinity | First Deriva- <br> tive at $t=0$ |
| :---: | :---: | :---: | :---: |
| (A) | $\pm\left[\varepsilon^{t}-1\right]$ | Yes | $\pm 1$ |
| (B) | $\pm[\operatorname{Cosh}(t)-1]$ | Yes | 0 |
| (C) | $\pm[\operatorname{Cos}(t)-1]$ | No | 0 |
| (D) | $\pm \operatorname{Sinh}(t)$ | Yes | $\pm 1$ |
| (E) | $\pm \operatorname{Sin}(t)$ | No | $\pm 1$ |

Figure DN2-3

Therefore, the resulting form of $U(t)$, the only acceptable form, the only one that meets all of the requirements, is:
(DN2-15)
$U(t)= \pm[\operatorname{Cos}(t)-1]$
$\mathrm{U}(\mathrm{t})=0$
$t>0$
$t \leq 0$.

This is identical to the more usual and convenient form:

```
(DN2-16) U(t) = \pm [1-Cos(t)]
```

which will be used for $t>0$ in the following development.

