## SECTION 20

## A Model for the Universe (10) The "Cosmic Egg"

Near the beginning of this Part III - On the Mechanics of the Universe, in section 10-The Probable Beginning, the hypothesis as to the initiating event of the universe was developed. Since that section the consequences implicit in that Origin have been elaborated and correlated to the known physical universe that we perceive and in which we participate. That development having progressed sufficiently, it is now possible to close the circle of development by applying the developed Universal Physics to the Origin.

## Paraphrasing from that section on The Probable Beginning:

Thus the hypothesis is that the interruption [of what would otherwise have been an infinite duration of the primordial absolute nothing] that gave us our universe was the starting of an oscillation: present to us at a very high frequency, of the general [1 - Cosine] form, and subject to the requirement that the change from nothing to something was constrained by the impossibility of an infinity.

That analysis yielded an initial event, the Origin, as in Figure 20-1, below (the same as Figure 10-1(2)).


Figure 20-1
Reexamination of this wave form reveals two problems. One, that it is an immediate mutual annihilation, will be dealt with shortly below. Of concern now is that an infinite rate of change still remains; the envelope of the oscillation has an infinite rate of change at $t=t_{0}$ as can be seen in Figure 20-2, below, which displays the envelope.


Figure 20-2

Viewed in a mathematical or graphical sense without any consideration of the physical reality represented, the envelope discontinuity at $t=t_{0}$ is not a difficulty. The only quantity that actually exists and is varying is the overall $U(t)$. The envelope is merely our perception of a characteristic of the wave form. The actual varying quantity, per Figure 20-1, has no discontinuity at $t=t_{0}$ for the reasons presented in section 10 - The Probable Beginning and its detail notes.

However, looking at the situation in a physical sense rather than purely mathematically, it has been shown that such oscillations as depicted in Figure 201 have related effects which are called energy, mass and / or charge. That energy and mass are equivalent and interchangeable has already been treated. They each, both, are merely aspects of oscillations in the medium, the energy equivalent being the product of the frequency and the Planck constant $[W=h \cdot f]$ and the mass equivalent being the energy equivalent divided by the square of the speed of light $\left[m=W / c^{2}\right]$.

But, this energy / mass / charge / oscillation is something other than nothing. It is a physical reality that did not exist prior to the Origin. It can no more leap from zero to a finite non-zero amount than could the original (or any) $U(t)$ so leap. That infinite rate of change in the amount of energy / mass / charge / oscillation at $t=t_{0}$ is no more acceptable than was the infinite rate of change encountered in the original analysis of the probable beginning and it must be corrected by the same kind of reasoning as then pursued: the envelope, also, had to originate as a [1 - Cosine] form of oscillation, which is the only form that avoids an infinite rate of change and matches the requirements of the situation. The envelope oscillation continued for the same reason as did the original wave: it constitutes less change for it to continue, once it has started, than for it to further change or terminate.

That original envelope oscillation was at a lesser frequency than the original wave by the definition of a wave form envelope. If it were at a greater frequency then the roles (envelope and wave) would be reversed. If it were at the same frequency it would not act as an envelope and the infinity problem would remain. If we designate the envelope frequency as $f_{e n v}$ and the frequency of the wave oscillation within the envelope as $f_{\text {wve }}$ then the envelope would be of the following form.

$$
(20-1) \quad U_{e n v}=\left[1-\operatorname{Cos}\left(2 \pi \cdot f_{e n v} \cdot t\right)\right]
$$

The wave is, as before, of the form

$$
(20-2) \quad \mathrm{U}_{\mathrm{wve}}= \pm \mathrm{A}_{0} \cdot\left[1-\operatorname{Cos}\left(2 \pi \cdot \mathrm{f}_{\mathrm{wve}} \cdot \mathrm{t}\right)\right]
$$

and the envelope modulating the wave is then

$$
\begin{aligned}
(20-3) \quad \mathrm{U}(\mathrm{t}) & =\left[\mathrm{U}_{\mathrm{env}}\right] \cdot\left[\mathrm{U}_{\mathrm{wve}}\right] \\
& = \pm \mathrm{A}_{0} \cdot\left[1-\operatorname{Cos}\left(2 \pi \cdot \mathrm{f}_{\mathrm{env}} \cdot t\right)\right] \cdot\left[1-\operatorname{Cos}\left(2 \pi \cdot \mathrm{f}_{\mathrm{wve}} \cdot t\right)\right]
\end{aligned}
$$

The " $\pm$ " in the above expressions is to account for the oscillation being in both $+U$ and $-U$, of course, so that conservation is maintained. That wave form appears in Figure 20-3, on the following page.


Figure 20-3
But, what about conservation? Is the energy / mass in $+U(t)$ "positive" energy / mass and that in $-U(t)$ negative? In a sense "yes" and in a sense "no".

The "yes" stems from that neither the mass effect nor the energy effect is a "real" reality. The only reality is the oscillations; all else is our perception of the effects that are produced by the centers and their waves. Among those effects are what we have chosen to refer to as mass and energy. The only reality, the oscillation, consists of two equal and opposite oscillations that mutually maintain conservation.

The "no" stems from that energy and mass are quantities having a scale range from zero to positive values. There is no such thing as negative mass or energy. (Negative energy amounts are spoken of in physics discussions but they are not absolutely negative, only negative relative to some other defined energy. For example, the energy of an atom's orbital electrons is negative relative to the energy that they would have if free of the atom.) Photons and electromagnetic waves carry energy and it is always "positive". That is our perception of the effects that they produce. Actually, a photon deriving from a center-ofoscillation in $-U$ is $180^{\circ}$ out of phase with one deriving from a $+U$ center, and reflects in that sense the same conservation as the Original $+U$ and $-U$ oscillations. Likewise, E-M radiation from a positive particle's motion is $180^{\circ}$ out of phase with E-M radiation from a correspondingly moving negative particle.

However, the form of $U(t)$ of equation 20-3 and Figure 20-3 still does not resolve the problem of an infinite rate of change at $t_{0}$. The [1 - Cosine] envelope is itself an oscillation that begins at $t_{0}$ with a sudden step from zero to its full amplitude. Figure 20-3, above, shows the first 2 cycles of the envelope oscillation, which if only the envelope is considered, is a simple oscillation at the envelope frequency, even though visually, in the Figure, it is only the trace of the peaks of the overall complex oscillation. It is energy / mass / oscillation that begins suddenly in its full amount at $t_{0}$ just as, in Figure 20-1, the oscillation of equation 20-1 begins at $t_{0}$.

Whether an oscillation in the U-medium is:

- pure and simple as the proton and electron,
- moderately complex as the neutron (where without the envelope the oscillation is that of a proton so that the neutron's additional mass must derive from the effect of the neutron's envelope oscillation),
- a mere half cycle as is the photon (which, consequently has no rest mass but does have kinetic mass due to its energy / mass / oscillation),
or whatever, it is an energy / mass / oscillation that cannot instantaneously leap from zero to a finite value in a manner that violates conservation. (A photon does leap from zero instantaneously into existence at its full amplitude, but not in a manner violating conservation. The requisite energy comes from the changing orbit of the radiating electron.)

Therefore, it is again necessary to introduce yet another envelope of [1 - Cosine] form to prevent the infinite rate of change at $t_{0}$ in the prior envelope. That correction will in turn require still another such correction and so ad infinitum. An (apparently at this point) infinite string of envelopes thus results as a necessity of the situation. The resulting $U(t)$ then is
(20-4)

$$
\begin{gathered}
\mathrm{U}(\mathrm{t})= \pm \mathrm{A}_{0} \cdot \prod_{i=1}^{i=\infty}\left[\left[1-\operatorname{Cos}\left(2 \pi \cdot \mathrm{f}_{\mathrm{env}}^{\mathrm{i}}\right.\right.\right. \\
\cdot t)]] \cdot \ldots \\
\cdots \cdot\left[\left[1-\operatorname{Cos}\left(2 \pi \cdot \mathrm{f}_{\text {wve }} \cdot \mathrm{t}\right)\right]\right. \\
\text { where the }\lceil\| \text { symbol (a large } \pi, \text { Greek "p") } \\
\text { means the product of the indicated factors. }
\end{gathered}
$$

This tentative $U(t)$ could take a variety of forms depending upon the answers to the following issues.

- Since a [1 - Cosine] envelope is sufficient to prevent a $t_{0}$ infinity in all that "precedes" it (its "internal wave" whether a simple oscillation or a complex of that and prior envelopes) could not the wave factor of equation 20-4 be simply the Cosine without the constant "1-"?
- In fact, taking that reasoning further, could not each of the envelope factors except the "last" or "outer" envelope be also merely the cosine function without the constant "1-"?
- While an envelope frequency must be less than the frequency of the wave that it modulates so that the various $f_{\text {env }}$ must be less than $f_{\text {wve }}$, what about the relationships among the various envelopes? Must each be successively at a lower frequency than the prior one, or could they all be the same frequency, or how are they?
- How does (did) other than an infinite string of envelopes come about, or is (was) the string of envelopes infinite ?

As was commented upon in section 10 - The Probable Beginning, dealing here with an event so unreachable directly by present analysis the only path of investigation available is a process of: reasoning - tentative hypothesis -
model - model testing for correlation with the presently known universe. That procedure must be iterated until a model yielding full correlation with the real physics of our universe results.

The reasoning is as follows.
(1) While the foregoing reasoning, leading from an original wave to an original wave and its envelope and then to a second envelope and then a third and so on, is sequential, the event was instantaneous. Analogous to the manner in which a cosine function, which has an infinite set of derivatives (which are the means by which it avoids an infinite rate of change), springs "full blown" into existence rather than occurring as the function followed by the first derivative, then the second, and so forth; so the overall Original oscillation, $\pm U(t)$ with its infinite set of envelopes also had to spring "full blown" into existence, not appearing first with one, then a second, and so forth, envelopes.
(2) Only the "outer" or "last" envelope being of the [1 - Cosine] form is, indeed, sufficient to control the difficulty of an infinite rate of change at $t_{0}$. Since all of the "inner" envelopes and the wave being simple cosines rather than [1 - Cosine] forms is far simpler it would appear to be the more likely actual situation.
(3) All of the envelopes may be at the same frequency and that form is much simpler than one having a variety of envelope frequencies. There are two reasons for this.

First, the only quantity that is actually varying is the wave. The envelopes are merely the trace of the peaks of all that is within each envelope.

Second, if each envelope frequency must be different then each must be at least slightly smaller than the prior. With an infinite set of envelopes and only the frequency range from slightly less than that of the wave down to slightly above zero being available each successive envelope could only be at an infinitesimally lower frequency than its predecessor in any case. Infinitesimally less is essentially the same as identical.
(4) The unending series of successive derivatives of a cosine results nevertheless in a limited or closed form, the cosine. It can be represented by an infinite series of terms which, because each successive term is sufficiently less than the prior term, has a definite sum, the cosine (the series is convergent).

But, it would appear that the infinite series of envelopes of $U(t)$, while theoretically necessary, cannot exist in a real physical situation. There must be some kind of convergence to a definite, limited sum or form. Furthermore each additional envelope corresponds to an additional increment of energy / mass and there cannot be an infinite amount of that. Something had to set a finite limit on the number of envelopes.
(5) Each additional envelope factor in equation 20-4 results in a higher frequency content in the overall expression. That is, as each envelope is added the expansion of the exponentiated cosines expression into a sum of individual frequency cosine terms becomes longer and acquires higher frequency terms. (Table 20-6, further below, demonstrates that as a cosine is raised to successively higher integer exponents the highest frequency component in the expansion also increases correspondingly.) But, the oscillation could not have had an actual component at infinite frequency.

Considering sound waves propagating in a gas as an analogy, there is an upper limit to the frequency of sound that can be propagated. The limit is set in two ways. The wave length of the sound waves decreases as the frequency increases. When the wavelength becomes reduced to on the order of the size of the individual particles of the gas it cannot further reduce because the particles cannot subdivide.

Likewise, as the frequency increases the oscillatory motion of the gas particles must become more rapid. But the mass of the particles makes more rapid motion ever more difficult and the motion is ultimately limited by the speed of light. The nature of the medium in which sound waves exist inherently sets a limit on the propagation of sound waves in that medium.

It is reasonable that there be some aspect of the medium, which, as we know, already limits the speed of U-wave propagation to the speed of light, which aspect likewise sets a limit on the highest frequency / lowest wavelength U-waves that can propagate. That must be the case if for no other reason than to again avoid an infinity and as a result the series of envelopes, of factors in equation 20-4, was limited to some finite but quite large amount. The real universe Original $U(t)$ had an enormous set of envelopes but not an infinite set; they were cut off at some point. (Further analysis of this cutting off is presented later in this section.)

This reasoning yields a revised $U(t)$, the form of the Original oscillation, the Cosmic Egg, as equation 20-5, below. $N_{0}$ is the number of envelopes, all at the same frequency, $f_{e n v}$. The constants, the "1-" parts, have been eliminated from all but the "oth" envelope (the most infinite, the last or "outer", envelope), and that envelope does not appear in the expression because the number of envelopes cut off (for reasons yet to be developed) long before that point $(\infty)$.

The resulting form of $U(t)$, the "Cosmic Egg", is
$(20-5) \quad U(t)= \pm A_{0} \cdot \operatorname{Cos}^{N_{0}}\left[2 \pi \cdot f_{e n v} \cdot t\right] \cdot \operatorname{Cos}\left[2 \pi \cdot f_{\text {wve }} \cdot t\right]$
It turns out that whether the constant, the "1-" part of the envelopes, is present or not (other than that of the " $\infty$ th" one) the net effect on the form of the "Cosmic Egg" oscillation is the same because $N_{O}$ is so extremely large. This is demonstrated in Figures $20-4$ on the following page, which show the convergence of the two different wave forms, $[1-\operatorname{Cos}(x)]^{n}$ versus $[1-\cos (x)] \cdot \cos ^{n-1}(x)$, into the same wave form for moderately large $n$, $n=100$, which is still far smaller than $N_{0}$.

Figure 20-4 Comparison of $U(t)$ "Cosmic Egg" Wave Forms (Amplitude Normalized)


$$
\text { a. }[1-\operatorname{Cos}(x)]^{n} \text { For } n=1,3,10,100
$$


b. $[1-\operatorname{Cos}(x)] \cdot[\operatorname{Cos}(x)]^{n}$ For $n=0,2,9,99$

c. $[1-\operatorname{Cos}(x)]^{100}$ versus $[1-\operatorname{Cos}(x)] \cdot[\operatorname{Cos}(x)]^{99}$

Part (a) of the figure has the "1-" part in every factor whereas part (b) has the "1-" part in only one factor. As the exponent, $n$, increases the two wave forms increasingly resemble each other. At part (c) of the figure the two wave forms are almost identical for the exponent still only $n=100$.

For very large $n$, that is very large $N_{0}$ of equation 20-5, the converging of the wave form into a single narrow peak proceeds to a momentary "spike" per cycle. ( $N_{0}$ is found further below to be about $10^{84}$.) Figure 20-5, below, shows the appearance of the wave form for extremely large $n$, that is for $n=N_{0}--$ what the wave form of the Original "Cosmic Egg", the start of our universe, "looked like".

(20-5) U(t) $= \pm A_{0} \cdot \operatorname{Cos}^{N_{0}}\left[2 \pi \cdot f_{\text {env }} \cdot t\right] \cdot \operatorname{Cos}\left[2 \pi \cdot f_{\text {wve }} \cdot t\right]$
Figure 20-5
The $U(t)$ "Cosmic Egg" Wave Form
But, what is this $U(t)$; what kind of physical reality does it represent? It is an immense, complex, quasi-neutron, that is a neutron-like structure (because it is charge-neutral), a nucleus-like supercenter-of-oscillation. Its mathematical expression ultimately develops to be
(20-6)

$$
\mathrm{U}(\mathrm{t})= \pm 2 \cdot \mathrm{q} \cdot \cos ^{\mathrm{N}_{0}}\left[2 \pi \cdot \frac{\mathrm{f}_{\mathrm{p}}-\mathrm{f}_{\mathrm{e}}}{2} \cdot \mathrm{t}\right] \cdot \cos \left[2 \pi \cdot \frac{\mathrm{f}_{\mathrm{p}}+\mathrm{f}_{\mathrm{e}}}{2} \cdot \mathrm{t}\right]
$$

so that it is the source, the cause, of the value of $q$ and of the oscillation frequencies, $f_{e}$ and $f_{p}$, (and therefore of the masses since $h \cdot f=m \cdot c^{2}$ ) of the fundamental particles, the electron and the proton. The expression therefore involves all of the fundamental physical constants: $\pi, c, h$, and $q$, except $\delta$. This result develops as follows.

In section 17-A Model for the Universe (7) - The Atomic Nucleus - The Nuclear Species the general equation for the oscillation of an atomic nucleus supercenter-of-oscillation was developed, equation 17-2, repeated below. That equation is in terms of $Z, A$, and $N$, the atomic number, the atomic mass number and the number of nuclear neutrons for a particular nucleus.

## The General Nuclear Equation

(17-2)

$$
\begin{aligned}
\mathrm{U}\left[z^{S} y^{A}{ }^{A}\right] & =[\text { A protons }+[\mathrm{N}=\mathrm{A}-\mathrm{Z}] \text { electrons }] \\
& =\mathrm{U}_{\mathrm{C}} \cdot\left[\mathrm{Z}-\operatorname{Cos}\left[2 \pi \cdot \mathrm{~A} \cdot \mathrm{f}_{\mathrm{p}} \cdot \mathrm{t}\right]+\operatorname{Cos}\left[2 \pi \cdot \mathrm{~N} \cdot \mathrm{f}_{\mathrm{e}} \cdot \mathrm{t}\right]\right]
\end{aligned}
$$

Now, if we let $Z=0$ and $A=N$, then that equation becomes an expression for a complex nuclear supercenter that consists of $N$ neutrons, all colocated in a single "particle". It is a single complex center-of-oscillation that is a multi-neutron per equation $20-7$, below.

$$
\text { (20-7) } \begin{aligned}
& \underline{A n N-M u l t i p l e ~ N e u t r o n ~} \\
\mathrm{U}\left[S_{S y m}^{N}\right]= & {[\mathrm{N} \text { protons }+\mathrm{N} \text { electrons }] } \\
= & \mathrm{U}_{\mathrm{C}} \cdot\left[\operatorname{Cos}\left[2 \pi \cdot \mathrm{~N} \cdot \mathrm{f}_{\mathrm{e}} \cdot \mathrm{t}\right]-\operatorname{Cos}\left[2 \pi \cdot \mathrm{~N} \cdot \mathrm{f}_{\mathrm{p}} \cdot \mathrm{t}\right]\right]
\end{aligned}
$$

The above expression for a multiple neutron is, however, the difference of two cosines whereas $U(t)$ is the product of two cosines. That disagreement between the two forms is only one of the mode of expression, however. A simple trigonometric identity turns a sum or difference of two cosines into a product. Applying that formulation equation $20-8$, below, is obtained for a multiple neutron.

$$
\begin{aligned}
& \text { (20-8) } \mathrm{U}\left[{ }_{0} S y m^{N}\right]=2 \cdot \mathrm{U}_{\mathrm{C}} \cdot \operatorname{Cos}\left[2 \pi \cdot \mathrm{~N} \cdot \frac{\mathrm{f}_{\mathrm{p}}-\mathrm{f}_{\mathrm{e}}}{2} \cdot \mathrm{t}\right] \cdot \operatorname{Cos}\left[2 \pi \cdot \mathrm{~N} \cdot \frac{\mathrm{f}_{\mathrm{p}}+\mathrm{f}_{\mathrm{e}}}{2} \cdot \mathrm{t-} \mathrm{\Pi}\right] \\
& =2 \cdot \mathrm{U}_{\mathrm{C}} \cdot \operatorname{Cos}\left[2 \pi \cdot \mathrm{~N} \cdot \mathrm{f}_{\mathrm{env}} \cdot \mathrm{t}\right] \cdot \operatorname{Cos}\left[2 \pi \cdot \mathrm{~N} \cdot \mathrm{f}_{\mathrm{wve}} \cdot \mathrm{t}\right] \\
& \text { Where: (a) The }-\pi \text { is merely a phase angle and may } \\
& \text { be dropped as of no significance here. } \\
& \text { (b) } f_{e n v} \equiv \frac{f_{p}-f_{e}}{2} \quad \text { and } \quad f_{\text {wve }} \equiv \frac{f_{p}+f_{e}}{2}
\end{aligned}
$$

However, the $U(t)$ wave form, equation 20-5, involves a cosine raised to an exponent whereas there are no exponents in the above equation $20-8$. But the form of a cosine raised to an exponent can be expanded into a sum of terms having no exponents. Table 20-6, below, presents the expansion of the function $\cos ^{n}(x)$ for several values of $n$ and indicates the pattern of the expansion.

$$
\begin{aligned}
& \cos ^{1}(x)=1 \cdot[+\cos (x)] \\
& \cos ^{2}(x)=\frac{1}{2} \cdot[1+\cos (2 x)] \\
& \cos ^{3}(x)=\frac{1}{4} \cdot[+3 \cos (x)+\cos (3 x)] \\
& \left.\cos ^{4}(x)=\frac{1}{8} \cdot[3 \quad+\cos (2 x) \quad+4 x)\right] \\
& \cos ^{5}(x)=\frac{1}{2^{5-1}} \cdot[\cdots \text { etc. }]
\end{aligned}
$$

Table 20-6

## Expansion of Exponentiated Cosines

The $U(t)$ of equation 20-5 with its $\operatorname{Cos}^{N_{0}}$ term replaced with its expansion per Table 20-6, then becomes an expression for the sum of an immense number of various forms of super-neutrons as illustrated below. In effect it is essentially an enormous super-neutron, a particle that consists of an
immense number of neutrons joined together in the form of a type of atomic nucleus.

For example, using $N_{0}=5$ to illustrate the effect, the expansion of equation 20-5 per Table $20-6$ results in equation 20-9, below. (Of course $N_{0}$ is immensely greater than 5 and the actual expansion of the cosine to that power is enormously more complex)
Example U(t) (Simplified) Expansion
(20-9)

$$
\begin{aligned}
& \mathrm{U}(\mathrm{t})_{\mathrm{N}_{0}=5}= \pm \mathrm{A}_{0} \cdot\left[\operatorname{Cos}^{5}\left[2 \pi \cdot \mathrm{f}_{\mathrm{env}} \cdot \mathrm{t}\right]\right] \cdot \operatorname{Cos}\left[2 \pi \cdot \mathrm{f}_{\mathrm{wve}} \cdot \mathrm{t}\right] \\
& =\frac{ \pm \mathrm{A}_{0}}{16} \cdot\left[10 \cdot \cos \left[1 \cdot 2 \pi \cdot \mathrm{f}_{\mathrm{env}} \cdot \mathrm{t}\right]+\cdots\right. \\
& +5 \cdot \operatorname{Cos}\left[3 \cdot 2 \pi \cdot \mathrm{f}_{\mathrm{env}} \cdot \mathrm{t}\right]+\cdots \\
& \left.+1 \cdot \operatorname{Cos}\left[5 \cdot 2 \pi \cdot f_{e n v} \cdot t\right]\right] \cdot \operatorname{Cos}\left[2 \pi \cdot f_{\text {wve }} \cdot t\right] \\
& = \pm A_{0} \cdot\left[\frac{10}{16} \cdot \operatorname{Cos}\left[2 \pi \cdot f_{\text {env }} \cdot t\right] \cdot \operatorname{Cos}\left[2 \pi \cdot f_{\text {wve }} \cdot t\right]+\cdots\right. \\
& +\frac{5}{16} \cdot \operatorname{Cos}\left[6 \pi \cdot f_{e n v} \cdot t\right] \cdot \operatorname{Cos}\left[2 \pi \cdot f_{\text {wve }} \cdot t\right]+\cdots \\
& \left.+\frac{1}{16} \cdot \operatorname{Cos}\left[10 \pi \cdot f_{e n v} \cdot t\right] \cdot \operatorname{Cos}\left[2 \pi \cdot f_{\text {wve }} \cdot t\right]\right] \\
& = \pm A_{0} \cdot\left[\frac{10}{16} \cdot[\mathrm{~A} \text { Normal Neutron Form }]+\cdots\right. \\
& +\frac{5}{16} \cdot[\text { A Distorted Neutron-Like Form] }+\cdots \\
& +\frac{1}{16} \cdot[\text { A Distorted Neutron-Like Form] }]
\end{aligned}
$$

The above simple sample expansion of $\operatorname{Cos}^{N_{0}}$ illustrates two important aspects of the actual full $N_{0}$ expansion. The first is that the sum of the coefficients of the terms in the expansion, $10+5+1$ in the above example, always equals the divisor in front of the expression, 16 in the above example. As a result the expansion has the same overall amplitude as the unexpanded function (which would have to be so in any case if the expansion is mathematically valid).

For the $n=N_{0}$ case the number of coefficients is $\frac{1}{2} \cdot N_{0}$ and the divisor, always $2^{n-1}$, is $2^{N_{0}-1}$. But in spite of the immense numbers involved the overall oscillation has the same amplitude as that of all of the other centers of - oscillation that have been discussed. The above $A_{0}$ is the equation 20-8 $2 \cdot U_{C}=2 \cdot q$ (in equation 20-6).

The second aspect observed above is that all of the resulting terms in the expansion are in neutron-like form. The overall supercenter is an assembly of mostly distorted neutron-like components. However, the entire structure is based on the two frequencies $f_{w v e}$ and $f_{e n v}$ already found to be expressible as z/2. $\left[f_{p} \pm f_{e}\right]$.

More correctly stated the "Cosmic Egg", the $U(t)$, was a pair of such particles: an immense super-neutron and its anti-particle. The original $\pm U(t)$, the original oscillation that was the start of the universe, was the conservation maintaining pair.

Its complexity, which gave us the $N_{0}$ particles of our universe, resulted from the $N_{0}$ successive envelopes, all existing and acting simultaneously from the beginning of course. The envelopes themselves were essential in order to avoid an infinite rate of change, as already presented above. They were limited to $N_{0}$ in number by effects to be analyzed shortly below. $N_{O}$ is estimated later in this section to be on the order of $10^{84}$, a truly vast number in any case but far less than infinite.

This discussion of $U(t)$, the original oscillation the start of which was the start of the universe, has dealt so far only with the problems of the Origin, the problems of the transition from nothing to something. The something was, of course, the first instant of the entire universe. As such it must have contained in itself all of the mass / energy and all of the positive and negative charge of the universe.

That "Cosmic Egg", which gave birth to all that now is, had to fulfill the requirements that:

- initially it had to be a pair of equal and opposite oscillations (to maintain conservation and avoid "something from nothing"),
- it had to be the source of the universe's total mass / energy and charge,
- it had to be overall charge neutral (for the sake of conservation), and
- it had to be $+U /-U$ symmetrical (likewise to maintain conservation).

Thus, the first instant of the universe, the starting of the pair of oscillations, $\pm U(t)$, was, the moment that they started, the starting of the existence of a pair of complex supercenters-of-oscillation representing an immense number of neutrons and a matching immense number of antineutrons, the one in $+U$ and its anti-particle in $-U$. Each was a gigantic atomic nucleus that, when the two are taken together, contained all of the mass / energy of the universe in their immense mass and immense atomic mass number, $A$ which had the value $N_{0}$ in this case, and all of the positive and negative charge of the universe in the immense equal numbers of protons and electrons (in the one) and negaprotons and positrons (in the other) since each had the atomic number $Z=0$.

Each was unstable, of course; each was the most unstable nuclear structure that could be. They immediately decayed in an immense explosion of energy and particles, the event now called the "Big Bang".

Most probably the extreme instability and consequent immediate explosive decay account for the survival of the universe beyond its first moment, for otherwise the equal and opposite initial oscillations should have mutually annihilated. If that had happened it would have still been sufficient an event to
interrupt the otherwise infinite duration of nothing the avoidance of which was the cause of $U(t)$. Fortunately for us the "Cosmic Egg" radioactively decayed before it could annihilate. (There undoubtedly were numerous mutual annihilations of decay product particles.)

Referring to $U(t)$ as depicted in Figure 20-5, the so immediate decay undoubtedly occurred after only a minute portion, an infinitesimal portion, of the very first cycle had passed. It had to have been long before the first "spike". In that sense the initial event was very small, tenuous, hardly more than nothing because the instantaneous amplitude of $U(t)$ at that moment (the height of the curve above zero at that moment long before the first "spike") was also infinitesimal. It was hardly more than, essentially zero.

In that sense, the way that the universe started at all becomes a little more comprehensible. There was essentially almost no difference between "nothing", on-going absolute nothing, and the first infinitesimal moment of the original $U(t)$, the original oscillation.

Yet, it contained the entire universe.

## The Finite Limitation of the "Cosmic Egg" Envelopes

By "finite limitation" is meant that in the vicinity of the cut-off number of envelopes, $N_{0}$, the amplitude of each of the further successive envelopes being imposed on the Original $U(t)$ was successively significantly less than its immediate predecessor and the rate of that amplitude decrease increased sharply with further envelopes -- there was a sharp cut-off of amplitude. After a moderate number of such cut-off region envelopes the amplitude of any further envelopes had become infinitesimal. While such infinitesimal (and still continuing to become ever more infinitesimal) envelopes theoretically go on to an infinite number of them, even an infinite number of infinitesimal envelopes is not an infinity but, rather, is finite.

Two effects jointly contributed to there being such a sharp cut-off of the otherwise infinite number of Original "Cosmic Egg" envelopes. The first, and most important was a bandwidth effect. The second results from the mathematics of $U(t)$.

The bandwidth effect is exactly analogous to the bandwidth limitation found in electronic devices. An example is sound systems for human use. Such systems are unable to process signals of all frequencies because unavoidable capacitances and inductances in the devices set limits. Such devices always have bandwidths, ranges of frequencies that they can successfully process, which are determined by their components and design. Electronic sound systems are designed to match the abilities of human hearing. The human ear (itself biologically / mechanically bandwidth limited) experiences a sharp cut-off of its ability to detect sound at frequencies above 10,000 to $20,000 \mathrm{~Hz}$ (cycles per second) depending on the particular human. Electronic sound equipment is therefore economically designed for sounds up to that limit but not above.

In the case of the "Cosmic Egg" a similar bandwidth type of limitation operated. The analysis of that cut-off must be postponed until later in the next section after necessary preceding developments have been presented. Its effect and behavior was quite exactly analogous to electronic bandwidth limiting,
however. That natural bandwidth limiting cut-off frequency of the medium determined the value of $N_{0}$.

The second effect sharpened the cut-off; it made the falling off of amplitude much more drastic once it started. The key to that behavior is to be found in Table 20-6, earlier above, the expansion of the $\cos ^{n}(x)$ function.

The "Cosmic Egg" expression, equation 20-6, contains the factor

$$
\cos ^{N_{0}}\left[2 \pi\left(f_{\mathrm{env}}\right) t\right]
$$

and that factor creates the set of envelopes of the Original oscillation. The expansion of the cosine raised to the power of its $N_{0}$ exponent behaves according to the pattern illustrated in Table 20-6, of course. Analysis of the patterns in the coefficients of the individual terms of the $\operatorname{Cos}^{n}(x)$ expansion (the coefficients are the numbers that multiply the "Cos" functions) discloses a pattern related to the binomial expansion as demonstrated below.
(a) Binomial Expansion Coefficients $[a+b]^{n}$

(b) $\operatorname{Cos}^{n}(x)$ Expansion Coefficients

| $\mid \underline{n}$ |
| :---: |
| 0 |
| 1 |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| $\vdots$ |
| $:$ |
|  |

Coefficients Times $\operatorname{Cos}(*), *=0 x$ 1x 2 x 3x 4 x 5x $6 \mathrm{x} \quad 7 \mathrm{x}$ \begin{tabular}{r|rrrrrrr}
1 \& \& \& \& \& \& \& <br>

- \& 1 \& 1 \& \& \& \& \& <br>
1 \& - \& 1 \& \& \& \& \& <br>
- \& 3 \& - \& 1 \& 1 \& \& \& <br>
3 \& - \& 4 \& - \& - \& 1 \& \& <br>
- \& 10 \& - \& 5 \& - \& <br>
10 \& - \& 15 \& - \& 6 \& - \& 1 \& <br>
- \& 35 \& - \& 21 \& - \& 7 \& - \& 1 <br>
\end{tabular} $T_{i}=\frac{n!}{(n-i)!\cdot i!} \quad$ for $\quad[i=0$ to $i \leq 1 / 2 n]$

Table 20-7
Clearly, with the exception of the constant term (where, in the table, * $=0 x$ ) the other terms of the expansion of $\operatorname{Cos}^{n}(x)$ have the same coefficients as the corresponding terms of the binomial expansion. (Of course

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they must then be multiplied by $1 / 2 n-1$ per Table 20-6.) The formula for the binomial expansion can thus be used to obtain the coefficients for any value of $n$ in the expansion of $\cos ^{n}(x)$.

In the Original $U(t), N_{0}$ is the number of protons and electrons (as combined into neutrons) in the Original "Cosmic Egg" and that $N_{0}$, as the exponent of the envelope frequency cosine function, is the effective number of envelopes. The magnitude of that quantity, $N_{0}$, can be approximately determined. The procedure is to calculate the mass of the universe and divide it by the mass of an individual proton, which is

$$
(20-10) \quad \mathrm{m}_{\mathrm{p}}=1.67 \ldots \cdot 10^{-27} \text { kilograms. }
$$

Hydrogen atoms or their equivalent, that is protons and their associated electrons, are the vast majority, more than $99 \%$ of the matter of the universe. The electron is of negligible mass compared to the proton within the limited accuracy of the present calculation, so it is reasonable here to deem the mass of the universe as being all protons.

Determining the mass of the universe, $m_{U}$, proceeds by estimating the average mass density, $\rho$, and the applicable universe volume. The universe mass is then the product of the two and its determination by that procedure is developed fully in detail notes DN 13 - The Cosmos Now and Its Expansion From The Origin To The Present, which follows Section 21. From those detail notes the value to use for the mass density of the universe is their equation DN13-10:

$$
\text { (DN13-10) } \quad \rho_{U} \approx 5 \cdot 10^{-27} \mathrm{~kg} / \text { meter }^{3}
$$

Next the volume of the universe is needed so as to obtain the universe's mass as the product of the mass density and the volume. The volume of the universe develops as follows. The universe's radius applicable to the just obtained universe mass density should be based on an earlier time than the present because the investigations into estimating that density had to treat astral objects which we observe as they were some time in the past -- their distance from us divided by the speed of their light.

Those earlier times were in the range of 0 to 7 or 8 Gyrs into the past. As we look into the past at an increasing radial distance from us the observed volumes increase as that radius cubed. For that reason the applicable universe radius to use with the universe mass density just determined is that which existed at the time into the past $t \approx 6.5$ Gyrs ago. The development in detail notes $D N 13$, particularly Figure DN-4d, indicates that the estimated radius of the universe for the present calculation is:

$$
\text { (DN13-12) } \quad \begin{aligned}
\mathrm{R}_{\mathrm{U}} & =14 \mathrm{G}-\mathrm{Lt}-\mathrm{Yrs} \\
& =11 \cdot 10^{24} \text { meters. }
\end{aligned}
$$

Therefore the mass of the universe, as the product of its volume based on that radius and its equation DN13-10 density, is:

$$
\text { (DN13-13) } \begin{aligned}
\mathrm{m}_{\mathrm{U}} & =\rho_{\mathrm{U}} \cdot\left[4 / 3 \cdot \pi \cdot \mathrm{R}_{\mathrm{U}}^{3}\right] \\
& =3 \cdot 10^{49} \mathrm{~kg} .
\end{aligned}
$$

and the resulting value of $N_{O}$ is

$$
\text { (20-11) } \begin{aligned}
\mathrm{N}_{0} & =\frac{20-A \text { MODEL FOR THE UNIVERSE (10) - THE "COSMIC EGG" }}{\mathrm{m}_{\mathrm{p}}} \\
& =\frac{3 \cdot 10^{49}}{1.67 \cdot 10^{-27}} \\
& \approx 2 \cdot 10^{76}
\end{aligned}
$$

Analyses in recent years of the hypothesized or speculated likely scenario of the early universe, the "big bang", result in the rough estimate that there were then about 109, one billion, mutual annihilations for every proton present today. (This is based upon the observation that in the present day universe there are about 109 photons per proton. That estimate is a not unreasonable measure of the original number of annihilations. The mutual annihilations each produced two photons. Photons from other later causes, primarily black body radiation and electron orbital changes should be in an amount on the order of one photon per proton, far from the $10^{9}$, and leaving the Original mutual annihilations to account for that).

In that case the $2 \cdot 1076$ estimate for the present number of particles would give an Original $N_{0}$ value, at the initial instant before any mutual annihilations, of about $2 \cdot 1085$. While all of this estimating is quite approximate it would nevertheless be fairly reasonable to take that $N_{0}$ was on the order of 1085 .

That is an immense number. And, in this case it is the effective exponent of the envelope cosine in $U(t)$; it is the effective number of Original envelopes to the "Cosmic Egg". It is the bandwidth limit imposed by the very nature of the Original (and on-going) medium of U-wave oscillation and propagation.

Referring to Table 20-7(a), $N_{0}=1085$ is the $n$ of the formula. It is not practicable and most likely not possible to calculate all of the coefficients of the cosine expansion of the envelopes for 1085 envelopes. On the other hand, it is not unreasonable to calculate the 85 cases corresponding to the frequency multiples of the expansion: $101,102,103, \cdots 1085$, or to calculate some other representative sample.

Figure 20-8 on the next page is a plot of the relative magnitude of the successive coefficients of the various frequency multiples $(1 \cdot x, 3 \cdot x, \ldots$ $1085 \cdot x$ ), in the expansion of $\operatorname{Cos}^{n}(x)$ for $n=N_{0}=1085$. The plot indicates a sharp and drastic cut off, an attenuation of the higher frequencies. Figure 208(a) uses a linear horizontal axis and shows the cut-off in detail. Figure 20-8(b) uses a logarithmic horizontal scale to better present the tremendous range in frequency multiples from 1 to 1085 . It shows that the cut-off is quite sharp and drastic.

This cut-off is merely the action of the mathematics of $\cos ^{n}(x)$. The complete actual cut-off of the "Cosmic Egg" was the product of this cut-off and the bandwidth limitation discussed above and to be presented in the next section. If this effect operated in the case of an electronic sound system then, with increasing sound frequency, at the approach to the cut-off sound would suddenly cease rather than fade away in reducing amplitude as the bandwidth limitation, alone, causes.


Figure 20-8
The Cosn(x) Limitation of the "Cosmic Egg"
The cutting-off depicted in Figure 20-8, above can be calculated as follows. At the top of Figure 20-9, on the following page, the relative amplitude curve of Figure 20-8(a) is re-depicted. Immediately below it the slope, that is the rate of change, of that curve is depicted. Beneath that slope depiction is depicted the slope of that slope. And beneath that is depicted its slope.

The bottom curve in the figure, the expression for the rate of change of the rate of change of the rate of change (in differential calculus the third derivative) discloses the location of the "knee" of the cut-off region as being where that third rate of change equals zero. The mathematics of calculating those
several rates of change is only moderately awkward. It is presented at detail notes DN11 - Calculation of the $\operatorname{Cos}^{n}(x)$ Limitation at the end of this section.


Figure 20-9
Evaluating the Sharpness of the Cut-Off
The result of those calculations is that the "knee" occurs at the point that is $50 \%$ of the maximum. For example, if $N_{0}$ were $2 \cdot 1084$ then the "knee" of the curve would be at $1 \cdot 1084$ (not 1042).

## Other Aspects and Results of the Cosmic Egg

The overall development of $U(t)$ answers two questions that have remained unanswered (and not explicitly asked) throughout this discussion from the earliest sections of Part III - On the Mechanics of the Universe to the present one:

Why do the proton and the electron have the masses and the mass ratio to each other that they have?

Why for all of the basic centers-of-oscillation, the protons and the electrons, is the amplitude one single constant, $U_{C}$, which amplitude corresponds to the fundamental unit of charge, $q$, of the universe? Why is there a fundamental constant unit of charge, not subdividable, and multiples of which make up all other charges?

The answer is, of course, equations 20-5 and 20-7, the manner in which the universe came into existence in the form of one single immense and immensely complex particle the equivalent of $N_{0}$ neutrons. The frequency of the proton and of the electron, $f_{p}$ and $f_{e}$, and therefore their masses, are the mere chance outcome of the origin of the universe as it happened. The original wave had to have some frequency, $f_{\text {wve }}$, as did the original envelope, $f_{\text {env }}$. The values that they turned out to have were the mere chance outcome of the Origin as it happened, but they determined the values for the proton and the electron.

Similarly, the original oscillation had to have some amplitude. Its specific value is whatever it happened to be. The value that it turned out to have determined the fundamental charge of the universe, $q$.

Overall, $U(t)$ determined the total mass / energy and charge of the universe. The radioactive decay of the big bang and after had to meet the requirements for such decay as discussed in the earlier section 18-A Model for the Universe (8) - Radioactivity. That could only result in the family of specie with which we are familiar.

Why are there exactly two fundamental particles not one or several ? There had to be exactly two because there had to be the wave at some frequency and the envelopes at some other, lower, frequency.

Why are the electron and the proton stable whereas almost everything else appears to be unstable? 20th Century physicists have hypothesized that the proton is unstable and decays with a very long mean lifetime. Experiments have been conducted to detect this decay (by detecting an associated form of radiation called Cherenkov radiation). The experiments, although conducted under conditions and over periods of time that according to the physicists' theory should have detected proton decay events, have yielded no such events.

The proton and the electron are stable. The reason is that they are simple centers-of-oscillation; there is no simpler form to which they could decay and their structure is so simple that there is no imperative to attempt decay to a simpler form.

This leads directly to the subject of quarks because the quark hypothesis contends that protons and neutrons are composed of several quarks in combination. Quarks do not exist as fundamental particles. They are an ingenious theory, an attempt to extract some order out of the multitude of new and strange particles produced in high energy nuclear physics experiments. The particles produced in high energy particle accelerator collisions are real. They exist, albeit fleetingly, when produced by the collisions and at least some of them probably fleetingly existed during the Big Bang's radioactive decay of the "Cosmic Egg". But the quark is a well intended but incorrect fundamental particle hypothesis.

First, there is no place and no need for the quark in the now well validated Universal Physics. Furthermore, in spite of intense effort since the quark theory was proposed and became more or less accepted no free quark (other than perhaps a few cases of a "something" produced with immense collision energies and having a life time of less than a billionth of a second) has ever been detected and none ever will be. A "particle" so rare and of so brief a life time can hardly be deemed to be a "fundamental building block" of nature.

In addition, it can be observed that the quark hypothesis seems "unlikely". In fundamentals nature seems to tend to work in two's of things, not three's: two charge polarities, $"+$ " and " - "; two magnetic field poles, $N$ and $S$; two fundamental particles, proton and electron; two genders, male and female; two "universes of particles", particles and their anti-particles; and so on. The theory of the quark proposes that the proton and neutron are combinations of three quarks. It seems "unnatural" and quite difficult and unlikely. To successfully obtain combinations of two things, can be difficult enough, witness the problems we humans have with two genders and imagine our condition if there were three.

The proton and electron are fundamental particles as already described. It may well be that producing collisions among those particles at immense energies destroys or "blows apart the stable natural center-of-oscillation and does so in a manner producing quark-like pieces of residue. Such a result is evidence of the immense energy applied not the existence of quarks as fundamental particles. If one were to smash a large number of teacups one could observe that the resulting pieces are always concave surfaces and partial toroidal shapes (parts of the cup and the handle, of course). But it would be foolish to conclude from that that the fundamental components of teacups are such pieces.

A speculation with regard to the development of the universe is as follows. Traditional late 20th Century cosmology envisions the big bang as an explosion of light fundamental particles. The evolution of the heavier atomic specie is thought to have occurred within stars, after the formation of stars of course. But an alternative possibility, given that the starting particle was one immense nuclear-type structure in $+U$ and its anti-particle in $-U$, is that the initially explosive radioactive decay of those resulted in a substantial portion of the present incidence of the heavier elements as radioactive - decay - chain products. This speculation does not preclude heavy element formation in stars but it does offer a second source of the heavier elements, possibly the primary source.

Before closing this section it is appropriate to take up one other matter that is related to the "Cosmic Egg" and to the start of the universe, the important but, strangely, largely unrecognized problem of rotation. Why is there rotation? The universe is filled with rotation from the orbital electrons of atoms to moons, planets, star systems, galaxies and so forth. Of course it is apparent why rotation is necessary to the existence of the universe. The forces of attraction, electrostatic for atoms and gravitational for moons, planets, etc., would promptly collapse everything together were it not for the opposing effect of rotation producing, in effect, the centrifugal forces.

But, where did rotation come from? If the universe is filled with attractive forces how did they result in rotational systems rather than condensation of everything? If it all started with an immense outward explosion, the motions of which continue to this day, how did rotational systems evolve rather than mere straight line radial motion (radial from the source of the explosion).

In section 11-A Model for the Universe (1)-Field and Charge, it was observed that the "mutual" attraction or repulsion between charges is not really "mutual". The effect of each charge on the other is due to the arriving wave, which arrival occurs a short time after the wave departed its source center-ofoscillation. If the source center moved while the wave was traveling from it to the encountered center then the attraction / repulsion is not toward / away from where the source center is but where it was.

This fact that the Coulomb attraction between particles in motion is toward where they were, not where they are, naturally causes "attracting" particles to tend to take up rotary motion relative to each other rather than to move directly toward each other. Since gravitational attraction also involves the transit time of U-waves then gravitation, likewise, is an attraction to where the attracting body was, not where it is. Of course, in many cases the wave travel time is quite brief and the change of position of the source body may be quite
small. But, any deviation of the attraction away from being directly toward the center of the attracting body tends to produce rotary motion nevertheless.

This effect enabled our universe to develop as it did.

## Calculation of the $\operatorname{Cos}^{n}(x)$ Limitation

The objective is to calculate for the expansion of $\operatorname{Cos}^{n}(x)$ per Table 20-6 the portion of the total number of terms at which the "break" of Figure 20-8 occurs. That is, for the exponent $n$ with the terms being terms $1,2,3, \ldots . i, \ldots n$, what is the value of $i$ at the "knee" of the curve?

The coefficients of the terms in the expansion of $\cos ^{n}(x)$ were presented in Table 20-7(b) as they relate to the binomial expansion. For example the coefficients for $n=7$ are there given as

$$
\begin{array}{llll}
35 & 21 & 7 & 1
\end{array}
$$

Because only the coefficients toward the end of such series, where the falling-off in relative magnitude becomes important, are of interest and because the number of terms becomes too unmanageably large for large $n$, the analysis will deal with the terms in reverse order. That is, the above series would be presented as:

| $i:$ | 1 | 2 | 3 | 4 |
| ---: | ---: | ---: | ---: | ---: |
| $\mathrm{~T}_{\mathrm{i}}:$ | 1 | 7 | 21 | 35 |

where $i$ refers to which of the terms $1,2,3, \ldots$ i, ... $n$ is dealt with and $T_{i}$ is that term.
$S_{i}$ is $\Delta T_{i} / \Delta i$ the incremental rate of change from term to term. It corresponds to the first derivative. Since $\Delta i$ is always one then

$$
\text { (DN11-1) } \quad \mathrm{S}_{\mathrm{i}}=\left[\mathrm{T}_{\mathrm{i}+1}-\mathrm{T}_{\mathrm{i}}\right] / 1=\mathrm{T}_{\mathrm{i}+1}-\mathrm{T}_{\mathrm{i}}
$$

$R_{i}$ is $\Delta S_{i} / \Delta i$ the incremental rate of change from term to term of $S_{i}$. It corresponds to the second derivative. Since $\Delta i$ is always one then
(DN11-2) $\quad R_{i}=S_{i+1}-S_{i}$
$Q_{i}$ is $\Delta R_{i} / \Delta i$ the incremental rate of change from term to term of $R_{i}$. It corresponds to the third derivative. Rince $\Delta i$ is always one then

$$
\text { (DN11-3) } \quad Q_{i}=R_{i+1}-R_{i}
$$

Some values of these quantities (except for $\ell_{i}$ ) are as given in Table DN11-1, below.

$$
\begin{aligned}
& 11 \\
& 2 \mathrm{n} \quad \mathrm{n}-1 \\
& 3 \frac{n \cdot(n-1)}{1 \cdot 2} \quad \frac{n \cdot(n-3)}{1 \cdot 2} \quad \frac{n \cdot(n-3)-2 \cdot(n-1)}{1 \cdot 2} \\
& 4 \frac{n \cdot(n-1) \cdot(n-2)}{1 \cdot 2 \cdot 3} \quad \frac{n \cdot(n-1) \cdot(n-5)}{1 \cdot 2 \cdot 3} \quad \frac{n[(n-1) \cdot(n-5)-3 \cdot(n-3)]}{1 \cdot 2 \cdot 3} \\
& i \frac{n!}{(n-i)!\cdot i!} \quad \frac{n!\cdot(n-2 \cdot i+1)}{(n-i+1)!\cdot i!} \quad \frac{n!}{(n-i+2)!\cdot i!}\left[\begin{array}{c}
(n-i+2) \cdot \\
(n-2 \cdot i+1)- \\
i \cdot(n-2 \cdot i+3)
\end{array}\right]
\end{aligned}
$$

## Table DN11-1

By expanding the formulation in the large brackets into a quadratic expression $R_{i}$ can also be expressed as

$$
\left.\begin{array}{c}
(D N 11-4) \\
R_{i}=\frac{n!}{(n-i+2)!\cdot i!} \cdot\left[n-\frac{4 \cdot i-3 \pm[8 \cdot i+1]^{1 / 2}}{2}\right.
\end{array}\right] \begin{aligned}
& \text { (Product } \\
& \text { of two }_{\text {factors) }}^{\text {factor }}
\end{aligned}
$$

From equation DN11-3 $\Omega_{i}$, the third derivative, is obtained as

$$
\begin{aligned}
&(D N 11-5) \\
& Q_{i}= \frac{n!}{(n-i+2)!\cdot i!} \cdot\left[n-\frac{5 \cdot i-3 \pm\left[i^{2}+14 \cdot i+1\right]^{1 / 2}}{2}\right]-\cdots \\
& \cdots-\frac{\left(i^{2}-i\right) \cdot(n-2 \cdot i+5)}{i!}
\end{aligned}
$$

As indicated in Figure 20-9 and the associated text, the third derivative is zero at the "knee" of the curve of the various coefficients, where the sharp break is.

Setting $Q_{i}=0$, moving its second term to the other side of the equation and cancelling the common denominator factor of $i$ ! yields

$$
\begin{gathered}
\frac{n!}{(n-i+2)!}\left[n-\frac{5 \cdot i-3 \pm \pm\left[i^{2}+14 \cdot i+1\right]^{1 / 2}}{2}\right]=\frac{\left(i^{2}-i\right) \cdot(n-2 \cdot i+5)}{1} \\
{\left[n-\frac{5 \cdot i-3^{*} \pm\left[i^{2}+14 \cdot i+1\right]^{3 / 2}}{2}\right]=\left(i^{2}-i\right) \cdot(n-2 \cdot i+5) \frac{(n-i+2)!}{n!}} \\
412
\end{gathered}
$$

$$
\begin{aligned}
& {\left[5 \cdot i-3 \pm\left[i^{2}+14 \cdot i+1\right]^{1 / 2}\right] \quad \text { [The smallest "i" for } Q_{i}} \\
& \begin{array}{l}
\text { is } 4 \text {. Then }(n-i+2)=n-2 \text {. } \\
\text { But }(n-2)!/ n!=1 / n \cdot(n-1)
\end{array} \\
& \text { which approaches zero for } \\
& \text { large n.] } \\
& n=\frac{5 \cdot i-3 \pm\left[i^{2}+14 \cdot i+1\right]}{2} \\
& 2 \cdot n-5 \cdot i+3= \pm\left[i^{2}+14 \cdot i+1\right]^{1 / 2} \\
& {[2 \cdot n-5 \cdot i+3]^{2}=\left[i^{2}+14 \cdot i+1\right]} \\
& (24) \cdot i^{2}-(20 \cdot n+44) \cdot i+\left(4 \cdot n^{2}+12 \cdot n+8\right)=0 \\
& i=\frac{(20 \cdot n+44) \pm\left[16 \cdot n^{2}-608 \cdot n+1168\right]^{1 / 2}}{48} \\
& 20 \cdot n \pm\left[16 \cdot n^{2}\right]^{1 / 2} \quad[\text { For large "n" } 20 \cdot n \text { is much larger } \\
& =\frac{\text { than } 44 \text { and } 16 \cdot n^{2} \text { is much larger }}{48} \quad \text { than the balance of the radical.] } \\
& =\frac{20 \cdot n \pm 4 \cdot n}{48} \\
& =\frac{\mathrm{n}}{3} \\
& \text { [The squaring, introduced some few } \\
& \text { steps ago, produces an extraneous } \\
& \text { solution (the } \pm \text { taken as -).] } \\
& =\frac{\mathrm{n}}{2} \quad \text { [The non-extraneous solution which } \\
& \text { is (the } \pm \text { taken as +).] }
\end{aligned}
$$

