Gravitational Equivalent Frequency, the Planck Length and Matter Waves

by

Roger Ellman

Abstract

Analysis of gravitation and of matter waves discloses a greater significance than heretofore recognized for the frequency, wave, oscillation aspect of mass and matter as compared to the present emphasis on discrete particles.

The mass equivalency $m \cdot c^2 = h \cdot f$ applies to gravitational mass just as to inertial mass. The gravitational mass has a corresponding equivalent frequency, f. With that the significance of the Planck Length, l_{Pl}, clarifies; the Planck Length is fundamental to gravitation and, in effect, supersedes the Newtonian Gravitational constant, G, in that role. There is operational mechanical significance to the role of the Planck Length in gravitation whereas G is only a constant of proportionality.

The nature of relativistic kinetic energy and its significance for matter waves is developed. Theoretical implications are presented and would appear to imply a greater significance for the frequency, that is the wave, aspect of mass, matter, and particles in general than heretofore recognized.

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Gravitational Equivalent Frequency

Consider a small individual particle such as a proton or a neutron. The gravitational action of a massive body is the collective effect of the individual action, below, in each of its such particle components.

Newton's law of gravitation expressed in terms of m_{source} and $m_{acted\text{-}on}$ and with both sides of the equation divided by $m_{acted\text{-}on}$ is, of course,

$$(1) \qquad a_{grav} = G \cdot \left[\frac{m_{source}}{d^2}\right]$$

stating that gravitation is a property of a body's mass.

However, mass and energy are equivalent, so that a mass, m, is proportional to a frequency, f, that is characteristic of that mass. That is

$$(2) \qquad m \cdot c^2 = h \cdot f \qquad \text{or} \qquad f = [c^2/h] \cdot m$$

so that the m_{source} of equation *(1)* has a corresponding equivalent frequency, f_{source}.

That being the case, the gravitational acceleration, a_{grav}, can be expressed in terms of that frequency as the change, Δv, in the velocity, v, of the attracted mass per time period, T_{source}, of the oscillation at the corresponding frequency, f_{source}, as follows.

$$(3) \qquad a_{grav} = \Delta v \,/\, T_{source} = \Delta v \cdot f_{source}$$

Gravitation and the Planck Length

It can then be reasoned using equation *(3)* = equation *(1)* as follows .

$$(4) \qquad a_{grav} = \Delta v \cdot f_{source} = G \cdot \left[\frac{m_{source}}{d^2}\right]$$

Equation *(5)*, below, is obtained by using that frequency is proportional to mass so that with f_p and m_p as the proton frequency and mass then $f_{source} = [m_{source} / m_p] \cdot f_p$.

$$(5) \qquad \Delta v \cdot \left[\frac{m_{source}}{m_p}\right] \cdot f_p = G \cdot \left[\frac{m_{source}}{d^2}\right]$$

Rearranging and canceling m_{source} on both sides of the equation,

$$(6) \qquad \Delta v = \frac{G \cdot m_p}{d^2 \cdot f_p} \quad \text{per cycle of } f_{source}.$$

Then substituting, per equation *(2)*, $m_p = [\,h \cdot f_p] / c^2$,

$$(7) \qquad \Delta v = \left[\frac{G}{d^2 \cdot f_p}\right] \cdot \left[\frac{h \cdot f_p}{c^2}\right]$$

$$= \frac{G \cdot h}{d^2 \cdot c^2} \quad \text{per cycle of } f_{source}.$$

The Planck Length, l_{Pl}, is defined as

$$(8) \qquad l_{Pl} \equiv \left[\frac{h \cdot G}{2\pi \cdot c^3}\right]^{\frac{1}{2}} \qquad \text{so that} \qquad G = \left[\frac{2\pi \cdot c^3 \cdot l_{Pl}^{\,2}}{h}\right]$$

Substituting G as a function of the Planck Length from equation *(8)* into G as it is in equation *(7)*, the following is obtained.

$$(9) \qquad \Delta v = \left[\frac{2\pi \cdot c^3 \cdot l_{Pl}^{\,2}}{h}\right] \cdot \left[\frac{h}{d^2 \cdot c^2}\right]$$

$$= c \cdot \frac{2\pi \cdot l_{Pl}^{\,2}}{d^2} \quad \text{per cycle of } f_{source}.$$

This result states that:

- the velocity change due to gravitation, Δv,
- per cycle of the attracting mass's equivalent frequency, f_{source}, which quantity, $\Delta v \cdot f_{source}$, is the gravitational acceleration, a_{grav},
- is a specific fraction of the speed of light, c, namely the ratio of:
    - 2π times the Planck Length squared, $2\pi \cdot l_{Pl}^{\,2}$, to
    - the squared separation distance of the masses, d^2.

That squared ratio is, of course, the usual inverse square behavior.

This also means that at distance $d = \sqrt{2\pi} \cdot l_{Pl}$ from the center of the source, attracting mass, the acceleration, Δv, per cycle of that attracting mass's equivalent frequency, f_{source}, is equal to the full speed of light, c, the most that it is possible to be. In other words, at that [quite close] distance from the source mass the maximum possible gravitational acceleration occurs. That is the significance, the physical meaning, of l_{Pl} or, rather, of $\sqrt{2\pi} \cdot l_{Pl}$.

The physical significance of $\sqrt{2\pi} \cdot l_{Pl}$ is that it sets a limit on the minimum separation distance in gravitational interactions and it implies that a "core" of that radius is at the center of fundamental particles having rest mass. That is, equation *(9)* clearly implies that it is not possible for a particle having rest mass to approach another such particle closer than that distance.

That physical significance of $\sqrt{2\pi} \cdot l_{Pl}$, is so fundamental to gravitation and apparently to particle structure, that it more truly represents a fundamental constant than does l_{Pl}. For those reasons that length should replace l_{Pl} as a fundamental constant of nature as follows.

(10) The fundamental distance constant, δ

$$\delta^2 \equiv 2\pi \cdot l_{Pl}^2$$

$$\delta = 4.05134 \times 10^{-35} \text{ meters} \qquad \text{[2006 CODATA Bulletin]}$$

Equation *(9)* then becomes equation *(11)*.

(11) $$\Delta v = c \cdot \frac{\delta^2}{d^2} \quad \text{per cycle of } f_{source}.$$

a quite pure, precise and direct statement of the operation of gravitation. It states that gravitation is a function of the speed of light, c, and the inverse square law, in the context of the oscillation frequency, f_{source}, corresponding to the attracting, source body's mass.

The Wave Aspect of Gravitation and Particles

There is an implication in all of this that gravitation and the gravitational field involve something oscillatory in nature, traveling or propagating at c while oscillating at f_{source}. Essentially the same description can be made of light and of all electro-magnetic radiation. It would seem somewhat absurd for material reality to involve two different, overlapping such propagations. Rather, there must be one simple such underlying form for both effects, gravitational and electro-magnetic.

If the original definition of l_{Pl} had been in terms of h, not h-bar = $h/2\pi$, the distinction with regard to $\sqrt{2\pi}$ would not now be necessary. The 2π is a gratuitous addition to the statement of the Planck Length, and probably came about from the deeming of the cause of the Hydrogen atom's stable orbits as being quantization of orbital angular momentum.

The statement that the orbital electron's orbital angular momentum is quantized, as in

(12) $$m \cdot v \cdot R = n \cdot \left[h/2\pi\right] \qquad [n = 1, 2, \ldots]$$

is a mis-arrangement of

(13) $$2\pi \cdot R = n \cdot \left[h/m \cdot v\right] = n \cdot \lambda_{mw} \qquad [n = 1, 2, \ldots]$$

the statement that the orbital path, $2\pi \cdot R$, must be an integral number of matter wavelengths, λ_{mw}, long.

That mis-arrangement may have resulted from a lack of confidence in the fundamental significance of matter waves because of the failure to develop theory that produced acceptable, valid, matter wave frequencies, to give proper results in the obvious necessity that

(14) $$f_{mw} \cdot \lambda_{mw} = \text{particle velocity.}$$

A re-examination of Einstein's derivation of relativistic kinetic energy (which produced his famous $E = m \cdot c^2$) leads to a valid matter wave frequency as called for in equation *(14)*, as follows.

Einstein's Derivation of Relativistic Kinetic Energy

The relativistic kinetic energy, *KE*, classically or non-relativisticly = $\frac{1}{2}\cdot m\cdot v^2$, of a mass at some velocity, *v*, is equal to the work done by the force, *f*, acting on the particle or object of mass, *m*, over the distance that the force acts, *s*, to produce the velocity v and is calculated by integrating the action over differential distances as in Figure 1, below, but using relativistic mass, *m*, that is rest mass Lorentz contracted by its velocity. The result is then slightly processed as in Figure 2 ending in stating, for that development, Einstein's result that:

(15) $KE = m_V\cdot c^2 - m_r\cdot c^2$ = [Total Energy] – [Rest Energy]

[m_r is total mass at $v = 0$; m_V is total mass at $v \neq 0$].

$$KE = \int_0^S f\cdot ds \qquad \text{(definition)}$$

$$= \int_0^S \frac{d(m\cdot v)}{dt}\cdot ds \qquad \text{(Newton's 2}^{\text{nd}}\text{ law)}$$

$$= \int_0^{(m\cdot v)} \frac{ds}{dt}\cdot d(m\cdot v) \qquad \text{(Rearrangement of form)}$$

$$= \int_0^{(m\cdot v)} v\cdot d(m\cdot v) \qquad (v = {}^{ds}/_{dt})$$

$$= \int_0^v v\cdot d\left[\frac{m_r\cdot v}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}}\right] \qquad \frac{\begin{pmatrix}\text{m is } m_r \text{ Lorentz}\\ \text{contracted by v}\end{pmatrix}}{(m_r \text{ is rest mass})}$$

$$= \frac{m_r\cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r\cdot \int_0^v \frac{v\cdot dv}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} \qquad \begin{pmatrix}\text{integration}\\ \text{by parts}\end{pmatrix}$$

$$KE = \frac{m_r\cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r\cdot c^2\left[1 - \frac{v^2}{c^2}\right]^{1/2} - m_r\cdot c^2 \qquad [A] \begin{pmatrix}\text{integration}\\ \text{of 2}^{\text{nd}} \text{ term}\end{pmatrix}$$

Figure 1 – First Part

$$KE = \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \left[1 - \frac{v^2}{c^2}\right]^{1/2} - m_r \cdot c^2 \qquad [A]\,(\text{per figure 1})$$

$$= \frac{m_r \cdot v^2 + m_r \cdot c^2 - m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \qquad \left(\begin{array}{r}\text{place 2nd term}\\ \text{over 1st term}\\ \text{denominator}\end{array}\right)$$

$$= \frac{m_r \cdot v^2 + m_r \cdot c^2 - m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \qquad \left(\begin{array}{r}\text{expand term}\\ \text{within brackets}\end{array}\right)$$

$$= \frac{m_r \cdot c^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \qquad (\text{simplify})$$

$$KE = m_v \cdot c^2 - m_r \cdot c^2$$

Figure 2 – Second Part

[The appearance in this result that the energies are the product of the masses times c^2, the speed of light squared, was the origination of Einstein's famous $E = m \cdot c^2$. The concept falls out naturally from applying the Lorentz transforms to the classical definition of kinetic energy.

[It is somewhat surprising that Einstein was the first to do that inasmuch as it was Lorentz who developed the Lorentz transforms and the Lorentz contractions essential to the development.]

Alternative Treatment of the Same Derivation

If in the above original derivation one proceeds differently from Step [A] of figure 2 then there is a somewhat different result. This is done by moving the term $-m_r \cdot c^2$ to the left side of the equation, as $+m_r \cdot c^2$, then evaluating the three terms of the resulting equation as in Figure 3. The result is equivalent to

(16)

$$\begin{bmatrix}\text{Total}\\ \text{Energy}\end{bmatrix} = \begin{bmatrix}\text{Energy in}\\ \text{Kinetic Form}\end{bmatrix} + \begin{bmatrix}\text{Energy in}\\ \text{Rest Form}\end{bmatrix}$$

$$m_v \cdot c^2 = m_v \cdot v^2 + m_v \cdot \left[c^2 - v^2\right]$$

$$\mathrm{KE} = \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \left[1 - \frac{v^2}{c^2}\right]^{1/2} - m_r \cdot c^2 \qquad [\mathrm{A}]\ (\text{as the original})$$

$$\mathrm{KE} + m_r \cdot c^2 = \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \left[1 - \frac{v^2}{c^2}\right]^{1/2} \qquad (m_r \cdot c^2 \text{ moved})$$

Evaluating:

$$[1]\ \ \mathrm{KE} + m_r \cdot c^2 = \text{kinetic plus rest energies} = \text{total energy} = m_v \cdot c^2$$

$$[2]\ \ \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} = \begin{bmatrix} \text{A relativistically increased energy of} \\ \text{motion at velocity v, which increase} \\ \text{is zero when v = 0} \end{bmatrix} = m_v \cdot v^2$$

$$[3]\ \ m_r \cdot c^2 \cdot \left[1 - \frac{v^2}{c^2}\right]^{1/2} = \begin{bmatrix} \text{A relativistically reduced} \\ \text{"rest energy" which is the} \\ \text{at rest energy when } v = 0 \end{bmatrix}$$

$$= [1] - [2] = m_v \cdot c^2 - m_v \cdot v^2$$

Figure 3 – Alternative Treatment

The resulting energy equation is equation *(16)* repeated below.

(16)

$$\begin{bmatrix} \text{Total} \\ \text{Energy} \end{bmatrix} = \begin{bmatrix} \text{Energy in} \\ \text{Kinetic Form} \end{bmatrix} + \begin{bmatrix} \text{Energy in} \\ \text{Rest Form} \end{bmatrix}$$

$$m_v \cdot c^2 = m_v \cdot v^2 + m_v \cdot \left[c^2 - v^2\right]$$

Dividing that by c^2 to obtain an equation in mass, the result is equivalent to

(17)

$$\begin{bmatrix} \text{Total} \\ \text{Mass} \end{bmatrix} = \begin{bmatrix} \text{Mass in} \\ \text{Kinetic Form} \end{bmatrix} + \begin{bmatrix} \text{Mass in} \\ \text{Rest Form} \end{bmatrix}$$

$$m_v = m_v \cdot \left[\frac{v^2}{c^2}\right] + m_v \cdot \left[1 - \frac{v^2}{c^2}\right].$$

Why is the formulation for classical *Kinetic Energy, KE* = $\frac{1}{2} \cdot m \cdot v^2$, but *Energy in Kinetic Form* is simply $m \cdot v^2$ without the ½? When dealing with quite small velocities where relativistic effects are nil (v very small relative to c), the excursion of total energy above rest energy and the excursion of energy in rest form below rest energy are both minute and essentially linear. In that case the excursion above the rest case is essentially half of the total of the excursions above and below. The classical kinetic energy is then half, $\frac{1}{2} \cdot m \cdot v^2$, of the total energy in kinetic form, $m \cdot v^2$.

The Correct Matter Wave Frequency

Thus the traditional view of kinetic energy as the energy increase due to motion may not be valid as a description of the processes taking place. Before the encountering of the relativistic change in mass with velocity the traditional view did not lead to the problems that now appear when relativity is taken into account.

Using mass-in-kinetic-form and energy-in-kinetic-form to obtain a correct frequency of the matter wave proceeds as follows. The matter wave wavelength, λ_{mw}, as has been experimentally verified is

(18) $$\lambda_{mw} = \frac{h}{\text{particle momentum}} = \frac{h}{m \cdot v}.$$

Using traditional kinetic energy to obtain the matter wave frequency, f_{mw}, per equation *(2)* produces a matter wave velocity half the particle velocity, as equation *(19)*.

(19) $$f_{mw} = \frac{\text{kinetic energy}}{h} = \frac{\frac{1}{2} \cdot m \cdot v^2}{h}$$

$$v_{mw} = \lambda_{mw} \cdot f_{mw} = \left[\frac{h}{m \cdot v}\right] \cdot \left[\frac{\frac{1}{2} \cdot m \cdot v^2}{h}\right] = \frac{1}{2} \cdot v$$

Using energy-in-kinetic-form to obtain the matter wave frequency resolves the problem.

(20) $$f_{mw} = \frac{\text{energy in kinetic form}}{h} = \frac{m \cdot v^2}{h}$$

$$v_{mw} = \lambda_{mw} \cdot f_{mw} = \left[\frac{h}{m \cdot v}\right] \cdot \left[\frac{m \cdot v^2}{h}\right] = v$$

and the matter wave is traveling with and as the particle.

Because "energy in kinetic form" produces the correct matter wave frequency whereas "kinetic energy" does not validates the correctness of "energy in kinetic form" and its directly related "energy in rest form".

On that basis the wave aspect of matter is then established both experimentally (Davison and Germer and their successors) and theoretically (the above development). That gives new significance to the fact, observed at the time of Bohr's development of the relationship between atomic line spectra and atomic orbital structure, that the stable orbits of atomic electrons are an integer multiple of the orbital electron's matter wave length.

The fact of the stable orbits has long been accepted without a specific reason, a specific operative cause, for those orbits and only those orbits being stable. The matter wave of the orbiting electron now provides an operative reason, as follows.

For the orbit to be stable it must be the same for each pass, pass after pass. If each pass includes exactly an integer number of the orbital electron's matter wave lengths then each pass is identical to the others. But if, for example, the orbital path length contains only $^{9}/_{10}$ of a matter wave length, that is $^{9}/_{10}$ of the matter wave period, then the next pass will contain the missing $^{1}/_{10}$ of its prior matter wave length or wave period plus $^{8}/_{10}$ of the next, and so on. The matter wave being sinusoidal in form, the successive orbital passes will all differ from each other. Orbital angular momentum has nothing to do with orbit stability with regard to which it has no operative cause nor any operative effect.

Conclusion

Analysis of an aspect of gravitation discloses that there is a fundamental distance constant, δ, that plays a key role in gravitation.

The analysis also discloses that gravitational attraction is communicated from one gravitational mass [the attracting mass] to another gravitational mass [the attracted mass] by an oscillatory propagation, i.e. a wave propagation, at the speed of light, *c*. The velocity of the attracted mass toward the attracting mass is increased each cycle of the wave propagation by that fraction of *c* that is the squared ratio of the fundamental distance constant, δ, to the separation distance between the two masses.

That effect taking place each cycle of the attracting propagation, then the overall acceleration produced is directly proportional to the frequency of the attracting wave, which frequency is directly proportional to the attracting mass per the mass – energy relationship:

(2) repeated $$m \cdot c^2 = h \cdot f$$

with which the analysis began.

In addition the analysis has revised the significance of matter waves by resolving the problem of the matter wave frequency, a result of the corrected calculation of relativistic kinetic energy. The corrected status of matter waves then supports their application in resolving the operational cause of the atomic orbital electrons' stable orbits.

These results and factors together would indicate a greater significance for the frequency, wave, oscillation aspect of mass and matter as compared to the present emphasis on discrete particles in that regard.

References

1. R. Ellman, *The Origin and Its Meaning* (The-Origin Foundation, Inc., 2004, second edition) (available at http://www.The-Origin.org/download.htm)