SECTION 3

The Matter Wave Solution

The problem with matter waves was the failure to obtain a satisfactory matter wave frequency after DeBroglie developed the matter wave wavelength, a failure that resulted largely in abandonment of interest in matter waves. The solution to the problem is obtained from the relativistic calculation of kinetic energy.

EINSTEIN'S DERIVATION OF RELATIVISTIC KINETIC ENERGY

Kinetic energy, KE, is defined as the work done by the force, f, acting on the particle or object of mass, m, over the distance that the force acts, s. This quantity is calculated by integrating the action over differential distances.

(3-1)

$$KE = \int_0^s f \cdot ds \qquad \text{[Per above definition]}$$

$$= \int_0^s \frac{d(m \cdot v)}{dt} \cdot ds \qquad \text{[Newton's } 2^{nd} \text{ law]}$$

$$= \int_0^{(m \cdot v)} \frac{ds}{dt} \cdot d(m \cdot v) \qquad \text{[Rearrangement of form]}$$

$$= \int_0^{(m \cdot v)} v \cdot d(m \cdot v) \qquad [v = {}^{ds}/_{dt}]$$

$$= \int_0^v v \cdot d\left[\frac{m_r \cdot v}{\left[1 - \frac{v^2}{c^2}\right]^{\frac{1}{2}}}\right] \qquad \text{[m is } m_r \text{ Lorentz contracted by v. } m_r \text{ is rest mass]}$$

(3-1 continued)

$$= \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot \int_0^v \frac{v \cdot dv}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} \qquad \text{[Integration by parts]}$$

(3-2)

$$KE = \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \cdot \left[1 - \frac{v^2}{c^2}\right]^{1/2} - m_r \cdot c^2 \qquad \text{[Integration of 2nd term]}$$

$$= \frac{m_r \cdot v^2 + m_r \cdot c^2 \cdot \left[1 - \frac{v^2}{c^2}\right]}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \qquad \text{[Place 2}^{\text{nd}}\text{ term over 1}^{\text{st}}\text{ term denominator]}$$

$$= \frac{m_r \cdot v^2 + m_r \cdot c^2 - m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \qquad \text{[Expand term with brackets]}$$

$$= \frac{m_r \cdot c^2}{\left[1 - \frac{v^2}{c^2}\right]^{1/2}} - m_r \cdot c^2 \qquad \text{[Simplify]}$$

(3-3) $KE = m_v \cdot c^2 - m_r \cdot c^2$ [m_v is total mass at $v > 0$
m_r is total mass at $v = 0$
$m_v = m_r$ Lorentz transformed]

This result equation *(3-3)* states that:

{Kinetic Energy} = {Total Energy} - {Rest Energy}

or

{Total Energy} = {Kinetic Energy} + {Rest Energy}

The appearance in equation *(3-3)* that the energies are the product of the masses times c^2, the speed of light squared, was the origination of that concept, the famous Einstein's $E = m \cdot c^2$. The concept falls out naturally from applying the Lorentz transforms to the classical definition of kinetic energy. It is somewhat surprising that Einstein was the first to do that inasmuch as it was Lorentz who developed the Lorentz transforms and the Lorentz contractions.

ALTERNATIVE TREATMENT OF THE SAME DERIVATION

If in the above original derivation one proceeds onward differently from the first line of equation *(3-2)*, as below, a slightly different result is obtained.

(3-2 first line repeated)

$$KE = \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{½}} - m_r \cdot c^2 \cdot \left[1 - \frac{v^2}{c^2}\right]^{½} - m_r \cdot c^2$$

(3-5)

$$KE + m_r \cdot c^2 = \frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{½}} - m_r \cdot c^2 \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}$$

[Move the "$- m_r \cdot c^2$"]

Considering and evaluating the three terms of equation *(3-5)*:

(3-6) $KE + m_r \cdot c^2$ = Kinetic plus rest energies

= Total Energy

$= m_v \cdot c^2$

(3-7) $\frac{m_r \cdot v^2}{\left[1 - \frac{v^2}{c^2}\right]^{½}}$ = A relativistically increased energy of motion which equals zero when v = 0.

$= m_v \cdot v^2$

(3-8) $m_r \cdot c^2 \cdot \left[1 - \frac{v^2}{c^2}\right]^{½}$ = A relativistically reduced rest energy which equals the at rest energy when v = 0

= Equation *(3-6)* – Equation *(3-7)*

$= m_v \cdot c^2 - m_v \cdot v^2$

the result is that equation *(3-5)* is equivalent to

(3-9) [Total Energy] = [Energy in Kinetic Form] + [Energy in Rest Form]

$$m_v \cdot c^2 = m_v \cdot v^2 + m_v \cdot (c^2 - v^2)$$

and (dividing the above energy equation by c^2 to obtain an equation in mass)

(3-10) [Total Mass] = [Mass in Kinetic Form] + [Mass in Rest Form]

$$m_v = m_v \cdot v^2/c^2 + m_v \cdot (1 - v^2/c^2)$$

It is shown in Part IV, Section 6, that "in kinetic form" is real, and is an effect of the speed of light.

Why is the formulation for classical *Kinetic Energy* $KE = ½\cdot m\cdot v^2$ but *Energy in Kinetic Form* is simply $m\cdot v^2$ without the ½? When dealing with quite small velocities (v very small relative to c) the excursion of total energy above rest energy and the excursion of energy in rest form below rest energy are both essentially linear. In that case the portion above the rest case is essentially half of the total excursion above and below the rest case. The classical kinetic energy is then half, $½\cdot m\cdot v^2$, ½ times the total energy in kinetic form, $m\cdot v^2$, for $[v/c]$ quite small.

Application to the Problem of the Matter Wave

Thus the traditional view of kinetic energy as the energy increase due to motion may not be valid as a description of the processes taking place. Before the encountering of the relativistic change in mass with velocity the traditional view did not lead to problems in spite of its being an over-simplification.

Using mass- and energy-in-kinetic-form to obtain the frequency of the matter wave proceeds as follows.

$$(3\text{-}11)\qquad f_{mw} = \frac{m_V\cdot v^2}{h}\qquad \text{[equation } (2\text{-}6)\text{, but using } W_V\text{, energy-in-kinetic-form, for } W_k\text{, kinetic energy]}$$

Using this result for matter wave frequency and using the same relativistic mass, m_V, in equation *(2-5)* for the matter wavelength the velocity of the matter wave then is

$$(3\text{-}12)\qquad v_{mw} = f_{mw}\cdot\lambda_{mw} = \left[\frac{m_V\cdot v^2}{h}\right]\cdot\left[\frac{h}{m_V\cdot v}\right] = v$$

and the wave is traveling with and as the particle.

Application to the Atomic Electrons Stable Orbits

On that basis the wave aspect of matter is then established both experimentally (Davison and Germer and their successors) and theoretically (the above development). That gives new significance to the fact, observed at the time of Bohr's development of the relationship between atomic line spectra and atomic orbital structure, that the orbital lengths of the stable orbits of atomic electrons are an integer multiple of the orbiting electron's matter wave length.

The fact of the stable orbits has long been accepted without a specific reason, a specific operative cause, for those orbits and only those orbits being stable. The matter wave of the orbiting electron now provides an operative reason, as follows.

For the orbit to be stable it must be the same for each pass, pass after pass. If each pass includes exactly an integer number of the orbital electron's matter wave lengths then each pass is the same in that regard. But if, for example, the orbital path length contains only $^9/_{10}$ of a matter wave length, $^9/_{10}$ of the matter wave period, then the next pass will contain the missing $^1/_{10}$ of the matter wave length or wave period plus $^8/_{10}$ of the next, and so on. The matter wave being sinusoidal in form, the successive orbital passes will be all different.

It is this behavior which operatively causes the "stable orbits", and only those orbits, to be stable. It has nothing to do with angular momentum nor quantization of angular momentum. For the angular momentum hypothesis there is no underlying reason nor mechanism to produce stability or instability. The quantization of angular momentum concept is merely an invented defined condition, without operative cause, just as were the "stable orbits" it seeks to explain until their being here justified in terms of the operative matter wave behavior

The statement that the orbital electron's angular momentum is quantized, as in the following traditional equation

$$(3\text{-}13) \qquad m\cdot v\cdot R = n\cdot\frac{h}{2\pi} \qquad [n = 1,\ 2,\ \ldots]$$

is merely a mis-arrangement of

$$(3\text{-}14) \qquad 2\pi\cdot R = n\cdot\frac{h}{m\cdot v} = n\cdot\lambda_{mw} \qquad [n = 1,\ 2,\ \ldots]$$

a statement that the orbital path length, $2\pi\cdot R$, must be an integral number of matter wavelengths, $n\cdot\lambda_{mw}$, long. The latter statement has a clear, simple, operational reason for its necessity. The former statement is arbitrary and is justified only because it produces the correct result, even if without an underlying rational reason.

The assumption without any justification or support that the orbital electron’s angular momentum is quantized is part of the early foundations of Quantum Mechanics along with Einstein’s unjustified assumption that light is particulate in form for the photo-electric effect.