SECTION 8

Matter Waves

MATTER WAVES AND SPHERICAL CENTERS OF OSCILLATION

The matter wave traveling right along with the particle is like a kind of standing wave relative to the particle. A standing wave can be thought of as the sum result of two waves traveling in opposite directions through each other. If the frequencies and wavelengths are different then their interaction produces a new frequency called a "beat". The development of the beat is as follows.

The two waves are

(8-1) $$\text{Wave }\#1 = A\cdot\text{Sin}(2\pi f_1 t)$$

$$\text{Wave }\#2 = A\cdot\text{Sin}(2\pi f_2 t)$$

and the sum is

(8-2) $$\text{Wave Sum} = A\cdot\text{Sin}\left[2\pi f_1 t\right] + A\cdot\text{Sin}\left[2\pi f_2 t\right]$$

which by using a trigonometric equivalence can be arranged as

$$\text{Wave Sum} = 2A\cdot\text{Sin}\left[2\pi\frac{f_1+f_2}{2}t\right]\cdot\text{Cos}\left[2\pi\frac{f_1-f_2}{2}\right]$$

The cosine term frequency ½· [f1-f2] difference, is smaller than the sine term sum ½· [f1+f2]. If the expression is viewed as the higher frequency sine portion with the rest of the expression being the amplitude, as in equation 6-8, then

(8-3) $$\text{Wave Sum} = \left[2A\cdot\text{Cos}\left[2\pi\frac{f_1-f_2}{2}t\right]\right]\cdot\text{Sin}\left[2\pi\frac{f_1+f_2}{2}t\right]$$

$$= \left[\text{Varying Amplitude}\right]\cdot\text{Sin}\left[2\pi\frac{f_1+f_2}{2}t\right]$$

The wave form appears as in Figure 8-1, below.

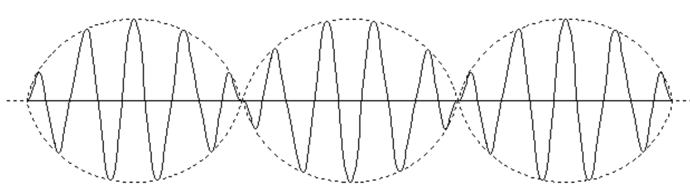

Figure 8-1

The solid-line curve in Figure 8-1 is the overall wave form. The dotted line, the *envelope*, is the varying amplitude. The overall wave form exhibits in the varying amplitude a periodic variation called the *beat*. The beat is real, not merely an appearance. For example two sound tones heard simultaneously produce an audible beat that one can hear. It is by listening to the beat that one tunes a piano or other musical instrument.

Matter waves are the beat that results from the *Spherical-Center-of-Oscillation*'s forward and rearward oscillations interacting with each other. This develops as follows. For a center in motion at velocity v.

(8-4) $\lambda_{fwd} = \lambda_v \cdot (1 - v/c) \qquad f_{fwd} = c/\lambda_{fwd}$

$\lambda_{rwd} = \lambda_v \cdot (1 + v/c) \qquad f_{rwd} = c/\lambda_{rwd}$

The beat frequency, using the "Varying Amplitude" portion of equation *6-8*, substituting f_{fwd} for f_1 and f_{rwd} for f_2, and then using equation *6-9*, is

(8-5)
$$f_{beat} = \frac{1}{2}\left[f_{fwd} - f_{rwd}\right] = \frac{1}{2}\left[\frac{c}{\lambda_v\left[1-\frac{v}{c}\right]} - \frac{c}{\lambda_v\left[1+\frac{v}{c}\right]}\right]$$

$$= \frac{c}{2\cdot\lambda_v}\cdot\left[\frac{\left[1+\frac{v}{c}\right]-\left[1-\frac{v}{c}\right]}{\left[1-\frac{v}{c}\right]^2}\right] = \frac{v}{\lambda_v}\cdot\left[\frac{1}{\left[1-\frac{v}{c}\right]^2}\right]$$

$$\lambda_{beat} = \frac{c}{f_{beat}} = \lambda_v \cdot\frac{c}{v}\cdot\left[1-\frac{v^2}{c^2}\right]$$

Substitute Eqn 4 – 2
$$\lambda_v = \lambda_r\cdot\frac{1}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}}$$

$$= \left[\lambda_r \frac{1}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}}\right]\cdot\frac{c}{v}\cdot\left[1-\frac{v^2}{c^2}\right]$$

Substitute per:
$$m\cdot c^2 = h\cdot f = h\cdot\frac{c}{\lambda} \rightarrow \lambda_r = \frac{h}{m_r\cdot c}$$

$$= \left[\frac{h}{m_r\cdot c}\right]\cdot\frac{c}{v}\cdot\left[\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}\right]$$

Substitute per Eqn 4 – 6
$$m_v = m_r\cdot\frac{1}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}}$$

$$= \frac{h}{m_v\cdot v}$$

which is the matter wavelength as previously obtained per equation *6-1* (in which the mass must be relativistic mass, m_V, of course). Thus matter waves are the beat that results from the *Spherical-Center-of-Oscillation*'s forward and rearward oscillations interacting with each other.

A moving center-of-oscillation as "seen" by an external observer appears as the waves propagated by the center in his direction appear. But, if one could, somehow, actually "see" the center itself pulsating as it does, the situation would be different. The interaction of the forward and rearward oscillations, which produce a beat at the matter wave frequency, are real. The effect is as follows (repeating the form of equations *6-6* through *6-8*, which were for any general oscillation, but now using the oscillations of a center-of-oscillation in motion).

(8-6) $\text{Forward Wave} = A\cdot\left[1+\text{Sin}(2\pi f_1 t)\right]$

$\text{Rearward Wave} = A\cdot\left[1+\text{Sin}(2\pi f_2 t)\right]$

[Note: $1 - \cos(x) \equiv 1 + \cos(180° - x)$
$\equiv 1 + \sin[90°-(180° - x)]$
$\equiv 1 + \sin(x - 90°)$
and the 90° phase is irrelevant, of course.]

and the sum is

(8-7) $\text{WaveSum} = A\cdot\left[2+\text{Sin}\left[2\pi f_1 t\right]+\text{Sin}\left[2\pi f_2 t\right]\right]$

Which again by using a trigonometric equivalence can be arranged as

$$\text{Wave Sum} = 2A + 2A \cdot \text{Sin}\left[2\pi \frac{f_1 + f_2}{2} t\right] \cdot \text{Cos}\left[2\pi \frac{f_1 - f_2}{2}\right]$$

The cosine term is at a lesser frequency than the sine term. If the expression for the wave sum is viewed as the (higher frequency) sine portion with the rest of the expression being the amplitude, as in equation 6-13, then

(8-8)

$$\begin{aligned}\text{Wave Sum} &= 2A \cdot \left[1 + \text{Cos}\left[2\pi \frac{f_1 - f_2}{2} t\right]\right] \cdot \text{Sin}\left[2\pi \frac{f_1 + f_2}{2} t\right] \\ &= 2A \cdot \begin{bmatrix} 1 + \text{cosine form of} \\ \text{Varying Amplitude} \end{bmatrix} \cdot \text{Sin}\left[2\pi \frac{f_1 + f_2}{2} t\right]\end{aligned}$$

In the case of a *Spherical-Center-of-Oscillation* $f_1 = f_{fwd}$ and $f_2 = f_{rwd}$. Likewise, A is U_C, the center average amplitude, the oscillation being of the form $U_C \cdot [1 - Cos(2\pi \cdot f \cdot t)]$.

The wave form appears as in Figure 8-2, below, for the forward-rearward interaction and the matter wave beat of the center's pulsation as it would be "seen" from the side relative to its direction of motion. Within the matter wave envelope is the center's spherical oscillation modified by the matter wave beat.

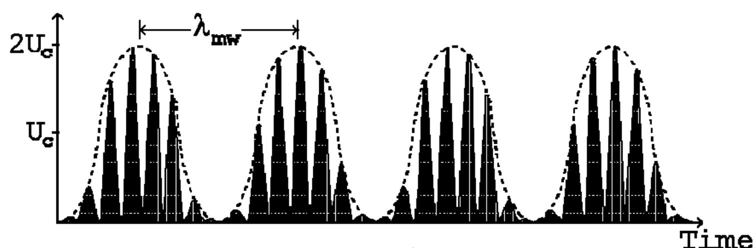

Figure 8-2
The Forward-Rearward Pulsation of a Center in Motion
Which is the Matter Wave