SECTION 14

A Model for the Universe (4)
Magnetic and Electromagnetic Field

When it is at rest, the interaction of a center-of-oscillation with its surrounding environment, the medium and other centers, is one simple effect, the electrostatic field and its Coulomb force. But, when the center is in motion, there is a variety of effects because of the variety of possible motions of the center. The principal such effects are: magnetic field, electromagnetic field (and electromagnetic radiation), and "matter waves", the "wave nature of matter". This section deals with the first two; matter waves are treated in the next section *15 - Quanta and the Atom*.

STATIC MAGNETIC FIELD

Static magnetic field is unchanging magnetic field and is an effect of a center or centers in motion at constant speed. The direction of the velocity may change but the speed must remain constant.

The direction of the force resulting from magnetic field was described in the earlier section *11 - Electric Field and Charge*. It is summarized in Figure 14-1, on the following page, in terms of the traditional 20th Century physics factors that are involved: a current (a flow of electric charges) with its associated magnetic field and a second current in motion relative to the first current's magnetic field. (The only direct result of such magnetic field interaction is a force acting on the charges that are moving relative to the magnetic field. However, traditionally that direct effect is usually described in terms of the consequent indirect effects of generated (induced) electric potential (voltage), current flow, force on and consequent motion of a conductor of current, force on a magnet, or whatever depending on the particular situation, an electric generator or motor, an electromagnet or solenoid, etc.).

Electric current being a flow of electric charges, one can speak of current or of some-quantity-of-charges-with-some-velocity. The following discussion must use both terminologies in order to relate the one to the other. A positive current in a given direction corresponds to positive charges flowing in that direction and equally corresponds to negative charges flowing in the opposite direction.

There are five basic cases of relative orientation of the two currents interacting. Any other situation can be resolved into some combination of some of the five basic cases as components. The five cases are as set out in Figure 14-1, below. In each case the analysis is of the effect of current #1, or rather the effect of its magnetic field, on current #2. Of course, exactly analogous reasoning would treat the effect of current #2 on #1. Both effects occur simultaneously just as in the earlier discussion of Coulomb's Law each center is in both source and encountered roles simultaneously even though the action is described in terms of only one of the roles at a time. I is the commonly used symbol for current. F_M is the magnetic force.

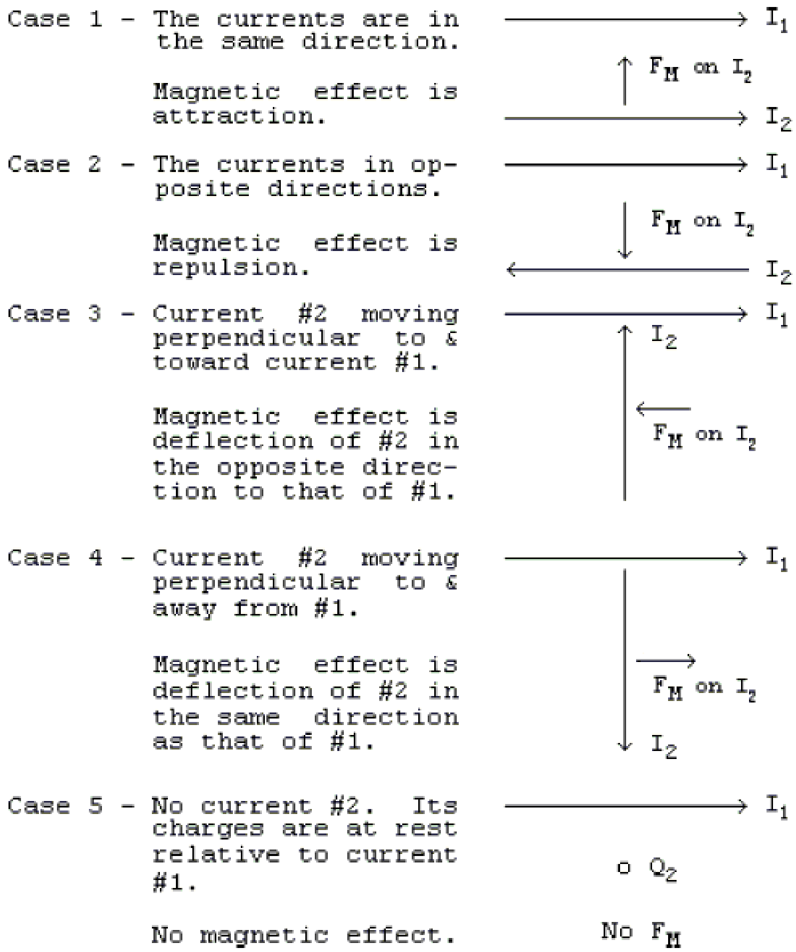


Figure 14-1

It is fortunate at this point that the problem of relativity has been resolved in favor of absolutivity. In the traditional terminology of relativity that "all motion is relative" if the velocity of the charges in I_1 and I_2 were the same there would be no way of distinguishing Case 1 from the case of both sets of charges being at rest, those charges comprising I_1 and those comprising I_2. Likewise, there would be no way of distinguishing Case 2 from Case 5, and so forth. While the electrical engineer will contend that that is no problem as he can distinguish between the cases because the currents involve associated electrical effects of electrical resistance, voltages, and so forth, such a contention is not valid for beams of particles in free space.

The magnetic effects are in addition to the electrostatic effects of the same charges. That is, the charges, the centers-of-oscillation whose motion is the current that produces the magnetic effects, have their natural electrostatic

(Coulomb) effect on each other when in motion as well as when at rest. However, that motion changes the amount of that effect and that change is the magnetic effect. To evaluate the magnetic force, then, it is necessary to examine the changes caused in the electrostatic force by the motion of those charges. If F_T is defined as the total interaction force, the combined effect of the electrostatic force, F_E, as it would be for those charges at rest and the magnetic force, F_M, due to their motion is, then

$$\mathbf{F_T} = \mathbf{F_E} + \mathbf{F_M} \tag{14-1}$$

where: F_T = the total interaction force between the charges,
F_E = the electrostatic force when the charges are at rest,
F_M = the magnetic force.

where the bold type indicates that the quantities have both magnitude and direction (are "vector" quantities) and both the magnitude and direction must be taken into account.

The analysis must therefore be: first an evaluation of the interactive force with the current's charges at rest, F_E; then an evaluation of F_T, the interactive force in the same configuration but with the charges in motion at velocity, v; and, finally, the comparison of those two results to obtain the magnetic effect, F_M.

Unfortunately, the natural geometry of the situations to be analyzed, the cases of Figure 14-1, makes the overall problem quite complicated. There is a great variety of configurations: the spherical electrostatic field of each charge, the cylindrical magnetic field of the currents and such currents in some cases perpendicular to each other, resulting magnetic forces in third directions, etc. As a result there is almost no way to obtain simple mathematical descriptions and analyses of what is actually happening. The real physical processes are direct and simple, but whether the mathematics is performed in rectangular, spherical or cylindrical coordinates some of the aspects of the problem will not be conveniently accommodated so that the mathematical expressions tend to become difficult.

In the terms of traditional 20th Century physics and taking the most simple situation, Cases 1 and 2 of Figure 14-1, it is obtained that the magnetic force between two parallel currents is of magnitude

$$F_M = \frac{\mu}{2\pi} \cdot \frac{I_1 \cdot I_2}{R} \cdot L \qquad \text{[Ampere's Law]} \tag{14-2}$$

where: L = the length of each of the two parallel current paths over which the force acts,
R = the distance between the two parallel paths,
μ = (Greek letter "mu") the permeability, a magnetic parameter of the substance between the two current paths (as already encountered in the discussion of the velocity of light).

Dividing by the path length, L, which is the length of current path I_1, the magnetic force of I_1 (the effect of its entire path length) on a unit length of I_2 (a minute length increment) is as in equation 14-3, below, which also is the corresponding effect of I_2 on a unit length of I_1.

$$F_{M,1} = \frac{\mu}{2\pi}\cdot\frac{I_1 \cdot I_2}{R} = F_{M,2} \qquad \text{[per unit length]} \tag{14-3}$$

For equations 14-2 and 14-3 to be valid the actual path length, L, must be much greater than the length of the region considered so as to prevent "end" effects. Theoretically the two current paths are infinitely long and a short section in the middle is being considered. That is, equation 14-3 expresses the magnetic effect of all of the magnetic field due to I_1 over its entire path length, L (where the path of I_1 is so long that the effect of the distant parts is negligible), on a unit length section of I_2, and vice versa.

Each of the currents, I_1 and I_2, in equation 14-3 is a stream of charges moving at velocity, v, as shown in Figure 14-2, below.

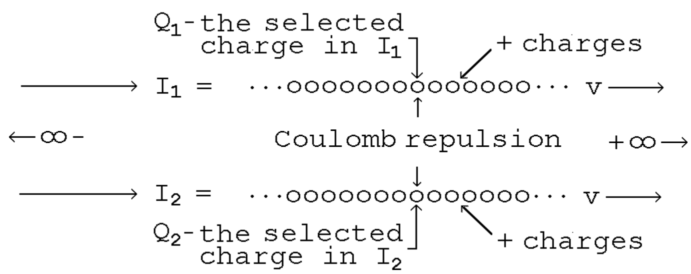


Figure 14-2

If Q_1 is the increment of charge in a unit length of the stream of charges that is I_1 and Q_2 is that for I_2, then, per Coulomb's law, the electrostatic force that would act on charge increment, Q_2, in I_2 due to the directly opposite charge increment, Q_1, in I_1 is of magnitude

$$F = \frac{Q_1 \cdot Q_2}{4\pi \cdot \varepsilon \cdot R^2} \qquad \text{[Coulomb's Law]} \tag{14-4}$$

Of course, as addressed below, Q_2 is similarly affected by all of the other charges in I_1, not just Q_1.

Since length increments and force per unit length are being treated, the increments of charge Q_1 and Q_2 must be replaced with charge-per-unit-length, ρ_1 and ρ_2, so that, when multiplied by length increments charge amount is obtained. The amount of charge, Q, located in length increment, dx, is

$$Q = \rho \cdot dx \tag{14-5}$$

To analyze the total electrostatic effect of all of the charges of which I_1 is composed on a single charge increment in I_2 the analysis of Figure 14-3, on the following page is pursued. In the figure R and $R(x)$ are radial distances between charge increments $Q = \rho \cdot dx$, that is R and $R(x)$ are the radial charge separation distance that appears in the denominator of Coulomb's law. The $dF(x)$ is the incremental Coulomb effect of a charge increment in I_1 on the selected charge increment in I_2. F_r is the peak value that $dF(x)$ attains (when its $[R(x)]^2$ in the Coulomb's law denominator is a minimum, R^2).

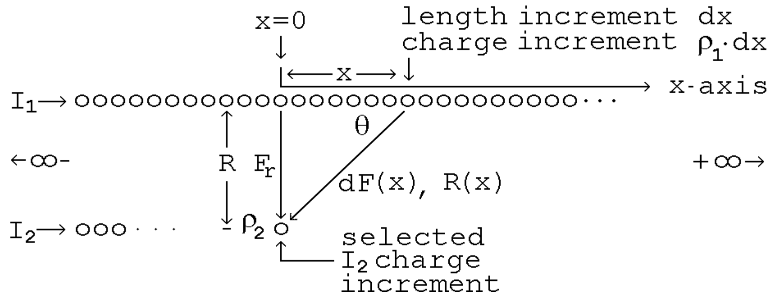


Figure 14-3

From Coulomb's law, the magnitude of $dF(x)$, the increment of the force, $F(x)$, exerted on the charge increment ρ_2 by $(\rho_1 \cdot dx)$, the amount of charge of I_1 that is located in length increment, dx, is

$$\text{(14-6)} \qquad dF(x) = \frac{(\rho_1 \cdot dx) \cdot \rho_2}{4\pi \cdot \varepsilon \cdot [R(x)]^2}$$

When $x=0$ so that $R(x)=R$ the rate of $dF(x)$ per dx is the pure "sideward" value, which equals the rest value and will be defined as F_r ,

$$\text{(14-7)} \qquad \frac{dF(x)}{dx} = \frac{\rho_1 \cdot \rho_2}{4\pi \cdot \varepsilon \cdot R^2} \equiv F_r$$

so that equation 14-6 then becomes

$$\text{(14-8)} \qquad dF(x) = F_r \cdot dx \cdot \frac{R^2}{[R(x)]^2}$$

From the right triangle $[\rho_2 - dx - x=0]$ in Figure 14-3, above, the sides of which triangle are R, x, and the hypotenuse, $R(x)$,

$$\text{(14-9)} \qquad R(x) = \sqrt{x^2 + R^2} \qquad \text{[law of Pythagoras]}$$

so that equation 14-8 becomes

$$\text{(14-10)} \qquad dF(x) = F_r \cdot \frac{R^2}{x^2 + R^2} \cdot dx$$

This force magnitude is directed diagonally to the lower left in Figure 14-3, above. That is, the charges in I_1 and I_2 are of the same sign and repel each other. The charge increment of I_1 at dx repels ρ_2 as shown in the figure. Depending on which charge increment of I_1 is considered, the angle at which the force increment, $dF(x)$, acts varies.

Consequently, the analysis must be broken down into two orthogonal components (see Figure 9-4). In terms of Figure 14-3 those components will be "horizontal", that is parallel to the figure's x-axis, and "vertical", at right angles to "horizontal". A quantity annotated with a horizontal arrow above it will mean that the "horizontal" component of the overall vector quantity is being treated. A quantity annotated with a vertical arrow to its left will mean that the "vertical" component of the overall vector quantity is being treated.

The magnitudes of the components, $\overrightarrow{dF}(x)$ and $\uparrow dF(x)$ relate to the magnitude of the overall vector quantity, $\mathbf{dF(x)}$, as

$$\text{(14-11)}\qquad \overrightarrow{dF}(x) = \mathbf{dF(x)}\cdot\frac{x}{R(x)} = \mathbf{dF(x)}\cdot\frac{x}{[x^2 + R^2]^{½}}$$

$$\uparrow dF(x) = \mathbf{dF(x)}\cdot\frac{R}{R(x)} = \mathbf{dF(x)}\cdot\frac{R}{[x^2 + R^2]^{½}}$$

Cases 1 and 2 (Figure 14-1)
The Static Force (The Charges at Rest), F_E

From $x=-\infty$ (immensely negative, to the left in Figure 14-3) to $x=0$ the horizontal components are all directed to the right. From $x=0$ to $x=+\infty$ (immensely positive, to the right in Figure 14-3) the horizontal components are all directed to the left. It can be seen that for all of the cases summed up from $x=-\infty$ to $x=+\infty$ the horizontal components will cancel out.

If the vertical components, $\uparrow dF(x)$, are summed over that range the result will be the total electrostatic force of the charges in I_1 on a single charge increment in I_2, the force being sought.

Substituting $dF(x)$, equation 14-10 for $\mathbf{dF(x)}$ in the expression for $\uparrow dF(x)$, equation 14-11, yields

$$\text{(14-12)}\qquad \uparrow dF(x) = F_r\cdot\frac{R^3}{[x^2 + R^2]^{1½}}\cdot dx$$

which increments must now be summed over the range from $x=-\infty$ to $x=+\infty$.

While it is not so yet, it will become necessary to treat the region to the left of Q_2, the $[x\leq 0]$ region, separately from the region to the right of Q_2, the $[x\geq 0]$ region. For that purpose the analysis is restated as two problems, one for the range $x=-\infty$ to 0 and one for the range $x=0$ to $+\infty$. Thus

$$\text{(14-13)}\qquad \sum_{x=-\infty}^{x=+\infty} = \sum_{x=-\infty}^{x=0} + \sum_{x=0}^{x=+\infty} \qquad [\Sigma \text{ means "summation"}]$$

The summing is done by the mathematical process called "integration", explained in detail notes *DN 5 - Integral Calculus (Mathematics of Summing Infinitesimals)* and is performed on equation 14-9 in detail notes *DN 6 - Integration Details for Magnetic Field Derivations, Part (1).*

From equation 14-12, and using the two regions per equation 14-13, where the ∫ means that integration is to be performed, the process and the result are given in equation 14-14 on the following page. The "±" signs come about because of the square roots in the expressions. The solution that yields $2\cdot R\cdot F_r$ rather than *zero* is selected because, of course, there is a non-zero Coulomb force in the situation analyzed and it is positive in the direction chosen for positive in the analysis (down in Figure 14-3).

(14-14)
$$\uparrow F_E = \int_{-\infty}^{0} \uparrow dF(x) + \int_{0}^{+\infty} \uparrow dF(x)$$
$$= \int_{-\infty}^{0} F_r \cdot \frac{R^3}{[x^2 + R^2]^{1½}} \cdot dx + \int_{0}^{+\infty} F_r \cdot \frac{R^3}{[x^2 + R^2]^{1½}} \cdot dx$$
$$= \pm R \cdot F_r \quad + \quad \pm R \cdot F_r$$
$$= 2 \cdot R \cdot F_r$$

By substituting in the value of F_r from equation 14-7, one obtains

(14-15)
$$\uparrow F_E = 2 \cdot R \cdot F_r = 2 \cdot R \cdot \frac{\rho_1 \cdot \rho_2}{4\pi \cdot \varepsilon \cdot R^2} = \frac{\rho_1 \cdot \rho_2}{2\pi \cdot \varepsilon \cdot R}$$

The ratio of the magnitudes, F_M/F_E, is the F_M of equation 14-2 divided by the F_E of equation 14-12.

(14-16)
$$\frac{F_M}{F_E} = F_M \cdot \frac{1}{F_E} = \left[\frac{\mu}{2\pi} \cdot \frac{I_1 \cdot I_2}{R}\right] \cdot \left[\frac{2\pi \cdot \varepsilon \cdot R}{\rho_1 \cdot \rho_2}\right]$$
$$= \mu \cdot \varepsilon \left[\frac{I_1 \cdot I_2}{\rho_1 \cdot \rho_2}\right]$$

Since current is charge flow per unit time, then

(14-17)
$$I_1 = \rho_1 \cdot v_1 \qquad \text{and} \qquad I_2 = \rho_2 \cdot v_2$$

where v_1 and v_2 are the velocities of the charges in I_1 and I_2. Using these and, letting $v_1 = v_2 = v$ for simplicity then

(14-18)
$$\frac{F_M}{F_E} = \mu \cdot \varepsilon \left[\frac{[\rho_1 \cdot v_1] \cdot [\rho_2 \cdot v_2]}{\rho_1 \cdot \rho_2}\right] = \mu \cdot \varepsilon \cdot v^2$$

is obtained. Finally recognizing that $c^2 = 1/\mu \cdot \varepsilon$, by substituting $1/c^2$ for $\mu \cdot \varepsilon$ and canceling the identical $\rho's$

(14-19)
$$\frac{F_M}{F_E} = \frac{v^2}{c^2} \qquad \text{or} \qquad F_M = \frac{v^2}{c^2} \cdot F_E$$

is the result.

Although this analysis was performed here only for Case 1, it is true for all 5 cases. The magnitude of the magnetic force is the same (for analogous values of the currents, etc.) in all of Cases 1 - 4, and is zero, of course, for Case 5. Likewise, the simplifying assumption that the velocity of the charges in each of the current flows was the same does not change the general validity.

The analysis so far, while developing a somewhat new result and taking a somewhat new point of view, is nevertheless entirely a result of and performed in terms of traditional 20th Century physics. The relationship, equation 14-19, expresses the magnitude of the change to the electrostatic field that produces the magnetic effect.

THE MAGNETIC FORCE (THE CHARGES IN MOTION), F_M

Now, with the above result in hand, it is necessary to investigate how this comes about from the actions of centers-of-oscillation. In the above analysis, while the velocity of the charges was indicated in the figure, the velocity was taken to be zero for F_E and the magnetic effect, F_M, was obtained from equation 14-2, the traditional Ampere's Law. Now the analysis that produced F_E must be performed again but with treating the charges as centers-of-oscillation and modifying $F(x)$ (and, of course, $dF(x)$) as appropriate to the behavior of centers-of-oscillation in motion at velocity v.

The magnetic force, alone, cannot be independently calculated. Rather, it can only be found by calculating the total interactive force with the charges in motion, F_T, and subtracting from that the portion that occurs when the charges are not in motion, the F_E just obtained above. That is, from equation 14-1,

$$\text{(14-20)} \qquad \mathbf{F_T - F_E = F_M}$$

The calculation of F_T with the charges in motion should produce a new result, different from F_E, in that it reflects the magnetic action. Since velocity is now also a variable the symbols $F(x)$ and $dF(x)$ will be replaced with $F(v,x)$ and $dF(v,x)$.

Subtracting F_E of equation 14-14 from this new F_T, the F_M portion can be obtained (taking account of direction, i.e. a vector subtraction by components). The magnitude of that F_M should be the same as obtained from Ampere's Law and that is most easily verified by taking the magnitude ratio F_M/F_E, which should be the same as equation 14-19, which was obtained using the methods of traditional 20th Century physics.

As developed in the preceding section *13 - Motion and Relativity*, the motion of a center at constant velocity results in changes in the propagated wave and in the center's own oscillation. Of interest here is the amplitude, the effective value of the charge, rather than the frequency and wave length as was the case in the prior section. It is the variation in the effective value of the charge, Q $(\rho \cdot dx)$, entering into the calculation of the net force effect per equation 14-6, that produces the change. Now, however, unlike the development in and following equation 14-6, Q is not a constant so that F_r is not constant and this variation due to velocity must be included in the expression for $F(x)$ (and $dF(x)$) as used in equation 14-14, above, namely the new quantity $F(v,x)$ (and $dF(v,x)$).

In the prior section it was shown that the forward wave and center amplitude are reduced by the factor $[1-{}^{v}/c]$ because of the forward propagation at $c' = c-v$ and that the rearward wave and center amplitude are analogously changed by the factor $[1+{}^{v}/c]$. It was also shown that the "throwing forward" of the forward wave by the center's velocity and the "negative" of that for the rearward wave changed the net force effect of the wave by a "force component" equal to $[{}^{v}/c] \cdot F_r$, positive in the forward direction and negative in the rearward direction (where F_r is the force delivered at the same distance but from the center at rest or to its side). These changes are summarized in Figure 14-4 which depicts the wave propagated by the source center, Q_1 in the present analysis, and Figure 14-5, which depicts the encountered center, Q_2 in the present analysis. The "force component" due to the center's velocity is F_{fc}. Since the force effect of the wave and the center is directly proportional to the charge, the effects developed in section *13 - Motion and Relativity* can be treated as changes to the effective force.

The Propagated Wave from the Source Center at Velocity v

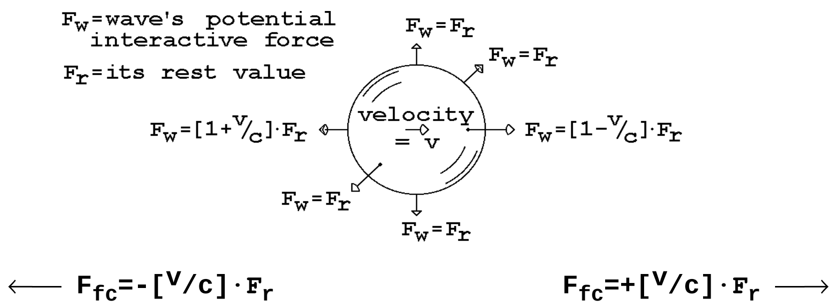


Figure 14-4

The Encountered Center's Oscillation at Velocity v

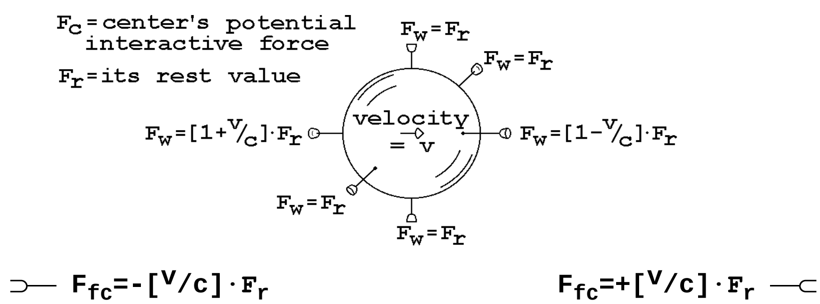


Figure 14-5

The amplitude of the wave or the center in a specific direction is the resultant of the sideward component with the forward or rearward component, as applicable. In motion the centers exhibit cylindrical symmetry around their direction of motion so that a two dimensional analysis will suffice for the following determination of the actual force effect in any particular direction.

Figure 14-6, below, illustrates the difference between conditions at rest and at velocity. At rest the centers' oscillation and waves have the same force effect in all directions. At velocity the force effect depends upon the angle of view relative to the direction of the velocity, angle θ in the figure.

The Centers of Figure 14-2 Enlarged

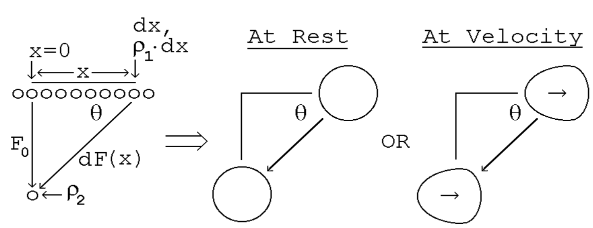


Figure 14-6

For F_{fc}, the "force component", the analysis of the effect of the "angle of view", θ, is simple. As shown in Figure 14-7, below, its magnitude in any direction is equal to the cosine of the angle between that direction and the pure forward direction times $[^v/c] \cdot F_r$. That relationship applies to both the F_{fc} of the wave and of the center (the point of view of Figure 14-4 and Figure 14-5, above).

The "Force component" Resultant When the Center is at Constant Velocity v

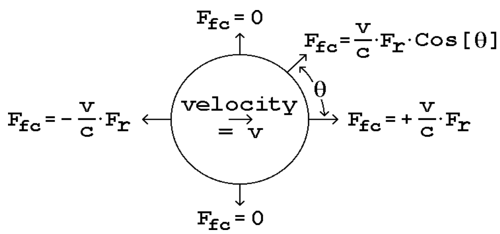


Figure 14-7

The treatment for the variation of the effect with the angle of view, θ, being essentially the same for both the wave and the center is also true for the principle effect of center velocity: the forward wave propagation at $c' = c - v$ producing a reduction of the forward force effect by the factor $[1 - ^v/c]$ and the analogous rearward behavior. The detailed development for the wave is as follows.

For the wave, for which the four two-dimensional components (using the cylindrical symmetry of the situation) are per Figure 14-4, the situation is not so simple as for the F_{fc}. If the velocity were zero then the wave resultant would be F_r in all directions and the model of it would be a circle as in Figure 14-8, below. In any non-orthogonal direction the force is obtained from the law of Pythagoras that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. In this case the force is the hypotenuse and the other two sides are its projection on the x- and y-axes.

The Wave Resultant When the Center is at Rest

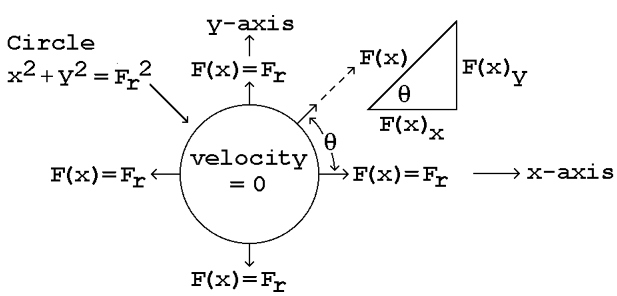


Figure 14-8

In the above figure, the center being at rest and its force effect being the same in all directions, $F(x)$ is always equal to F_r regardless of θ. However, when the center is in motion the situation is analogous but modified. With the center at velocity v, the circle must be modified into the combination of two ellipses, one for the forward direction and one for the rearward as in Figures 14-9 and 14-10, on the following page.

The Wave Resultant in the Forward Direction When the Center is at Velocity v

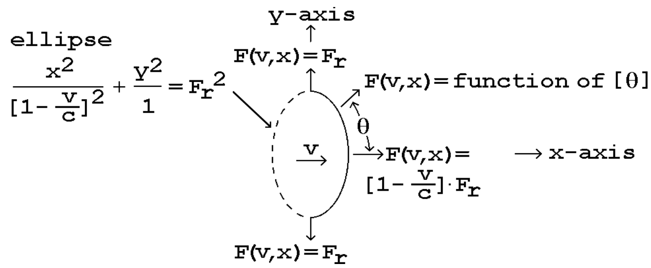


Figure 14-9

The Wave Resultant in the Rearward Direction When the Center is at Velocity v

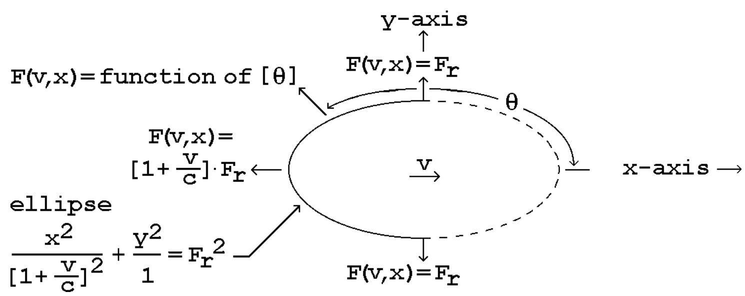


Figure 14-10

The equations of these two ellipses as given in the above figures can be generalized as

(14-21) $$\frac{x^2}{W^2}+y^2=F_r^2$$

$$\text{where:}\quad W \equiv 1-{}^{v}/_{c} \text{ for } +90° \geq \theta \geq -90°$$

and

$$W \equiv 1+{}^{v}/_{c} \text{ for } +90° \leq \theta \leq +270°$$

Changing equation 14-21 from its rectangular coordinates *(x,y)* to polar coordinates in the variables *(R,θ)*

(14-22) $$\frac{R^2\cdot \mathrm{Cos}^2\theta}{W^2}+R^2\cdot \mathrm{Sin}^2\theta = F_r^2$$

is obtained, and solving for the radius, *R*, the result is

(14-23) $$R = F_r\left[\frac{\mathrm{Cos}^2\theta}{W^2}+\mathrm{Sin}^2\theta\right]^{-½}$$

Using equation 14-23 to express *F(v,x)*, as defined in Figures 14-9 and 14-10, in terms of the direction angle, θ,

(14-24)
$$F(v,x) = F_r\left[\frac{\text{Cos}^2\theta}{W^2} + \text{Sin}^2\theta\right]^{-\frac{1}{2}}$$

where: $W \equiv 1-{}^{v}/c$ for $+90° \geq \theta \geq -90°$
and
$W \equiv 1+{}^{v}/c$ for $+90° \leq \theta \leq +270°$

The analysis returns now to the overall situation per Figure 14-11, below, which is the same as the *velocity=0* case of Figure 14-3 except that: the charges are now in motion, *F(x)* is *F(v,x)*, *dF(x)* is *dF(v,x)* and angle θ is defined in the figure below (and is functionally the same as in the above Figures 14-9 and 14-10).

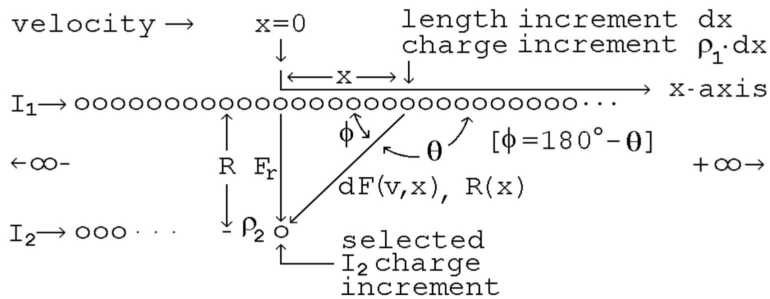


Figure 14-11

The following trigonometric relationships should be noted.

(14-25)
$$\text{Cos}(180°-\theta) = -\text{Cos}\theta \qquad \text{Cos}^2(180°-\theta) = \text{Cos}^2\theta$$
$$\text{Cos}(-\theta) = \text{Cos}\theta \qquad \text{Sin}^2(180°-\theta) = \text{Sin}^2\theta$$

The integration performed before, for the case in which the velocity was zero, was of equation 14-14, repeated below.

(14-14)
$$\uparrow F_E = \int_{-\infty}^{0} \uparrow dF(x) + \int_{0}^{+\infty} \uparrow dF(x)$$
$$= \int_{-\infty}^{0} F_r \cdot \frac{R^3}{[x^2 + R^2]^{1\frac{1}{2}}} \cdot dx + \int_{0}^{+\infty} F_r \cdot \frac{R^3}{[x^2 + R^2]^{1\frac{1}{2}}} \cdot dx$$

In that expression F_E is identical to F_T because the velocity is zero so that $F_M = 0$. Now, to deal with the charges in motion and non-zero velocity a new function, $f(v,x)$, is now defined:

(14-26)
$$f(v,x) = \frac{dF(v,x)}{dF(x)}$$

so that

(14-27)
$$dF(v,x) = f(v,x) \cdot dF(x)$$

and this function will be integrated in the expression of equation 14-14 by substituting $dF(v,x)$ per equation 14-27 for $dF(x)$. The result will be F_T, the total force at velocity v, rather than F_E, the static case force.

To proceed, from the definition of $f(v,x)$ per equation 14-26:

(14-28) $$f(v,x) = \frac{dF(v,x)}{dF(x)}$$ [Cases 1, 2, 5]

$$= \left[\frac{\text{Wave Resultant in direction } \theta \text{ at velocity } v}{\text{Wave Resultant in direction } \theta \text{ at } v=0}\right] \cdot \left[\frac{\text{Center Resultant in direction } [180°-\theta] \text{ at } v}{\text{Center Resultant in direction } [180°-\theta] \text{ at } v=0}\right] + \left[F_{fc} \text{ Wve toward } \theta \text{ at } v\right] - \left[F_{fc} \text{ Ctr toward } 180°-\theta \text{ at } v\right]^{*}$$

* At zero velocity there is no F_{fc} at all.

$$= \frac{\left[\frac{\cos^2\theta}{A^2} + \sin^2\theta\right]^{-½}}{1} \cdot \frac{\left[\frac{\cos^2\theta}{B^2} + \sin^2\theta\right]^{-½}}{1} + \left[C\cdot\cos\theta - D\cdot\cos\theta\right] \quad **$$

** Per equation 14-26, $f(v,x)$ is a measure of relative effects and does not include F_r.

Equations 14-25 have already been applied, and:

A = wave amplitude function of velocity
= $[1-{}^{v}/c]$ forward and $[1+{}^{v}/c]$ rearward

B = center amplitude function of velocity
= $[1-{}^{v}/c]$ forward and $[1+{}^{v}/c]$ rearward

C = wave F_{fc} function of velocity
= $[+{}^{v}/c]$ forward and $[-{}^{v}/c]$ rearward

D = center F_{fc} function of velocity
= $[+{}^{v}/c]$ forward and $[-{}^{v}/c]$ rearward.

Using equation 14-9 for $R(x)$ and the right triangle geometry of Figure 14-11, equation 14-28 becomes

(14-29) $f(v,x) =$ [Cases 1, 2, 5]

$$= \left[\frac{x^2/R(x)^2}{A^2} + \frac{R^2}{R(x)^2}\right]^{-½} \cdot \left[\frac{x^2/R(x)^2}{B^2} + \frac{R^2}{R(x)^2}\right]^{-½} + (C + D)\cdot\frac{x}{R(x)}$$

$$= \frac{A\cdot B\cdot(x^2 + R^2)}{[x^4 + (A^2 + B^2)\cdot R^2\cdot x^2 + A^2\cdot B^2\cdot R^4]^{½}} + \frac{(C + D)\cdot x}{(x^2 + R^2)^{½}}$$

values of A, B, C, and D for which are given in the following Figure 14-12 for Cases *1*, *2* and *5* cited in Figure 14-1, those for currents I_1 and I_2 parallel (in the same and in opposite directions) and that for $I_2 = 0$.

Referring back to equation 14-13, which separates the entire range to be calculated, that from $-\infty$ *to* $+\infty$, into two ranges, $-\infty$ *to* 0, *[<0]*, and 0 *to* $+\infty$, *[>0]*, the Range column in the figure below refers to those two ranges.

Case		Range	A	B	C	D
1	→	<0	$[1-{}^{v}/c]$	$[1+{}^{v}/c]$	$+{}^{v}/c$	$-{}^{v}/c$
	→	>0	$[1+{}^{v}/c]$	$[1-{}^{v}/c]$	$-{}^{v}/c$	$+{}^{v}/c$
2	→	<0	$[1-{}^{v}/c]$	$[1-{}^{v}/c]$	$+{}^{v}/c$	$+{}^{v}/c$
	←	>0	$[1+{}^{v}/c]$	$[1+{}^{v}/c]$	$-{}^{v}/c$	$-{}^{v}/c$
5	→	<0	$[1-{}^{v}/c]$	1	$+{}^{v}/c$	0
	·	>0	$[1+{}^{v}/c]$	1	$-{}^{v}/c$	0

Figure 14-12

The expression to be integrated now is

$$\textit{(14-30)}\qquad \uparrow F_T = \int_{-\infty}^{0} \uparrow dF(v,x) + \int_{0}^{+\infty} \uparrow dF(v,x)$$

$$= \int_{-\infty}^{0} \uparrow f(v,x)\cdot dF(x) + \int_{0}^{+\infty} \uparrow f(v,x)\cdot dF(x)$$

$$= \int_{-\infty}^{0} \left[\left[\frac{A\cdot B\cdot(x^2+R^2)}{[x^4+(A^2+B^2)\cdot R^2\cdot x^2 + A^2\cdot B^2\cdot R^4]^{1/2}} + \cdots\right.\right.$$

$$\left.\left.\cdots + \frac{(C+D)\cdot x}{(x^2+R^2)^{1/2}}\right]\cdot\left[F_r\cdot\frac{R^3}{[x^2+R^2]^{1\frac{1}{2}}}\right]\right]\cdot dx + \cdots$$

$$\cdots + \int_{0}^{+\infty}\left[\begin{array}{c}\text{The same above entire}\\ \text{expression a second}\\ \text{time}\end{array}\right]\cdot dx$$

where the two ranges are integrated separately because of the different values of A, B, C and D in the two ranges per the above Figure 14-12. Since the form of the two integrals in equation 14-30 is the same, only one of them need be followed through the integration process up to the point of inserting the limits to evaluate the integral. The integration and evaluation, which, unfortunately are quite complicated, are at detail notes *DN 6 - Integration Details for Magnetic Field Derivations, Part (2).*

The result of that integration is

$$\textit{(14-31)}\qquad \uparrow F_T = R\cdot F_r\cdot\Big[[A\cdot B + C + D]_{>0} + [A\cdot B + C + D]_{<0}\Big]$$

which is the result from the static case, $R \cdot F_r$ in each range, multiplied by an expression that takes account of the effect of velocity on the centers and their propagated waves. Per equation 14-20, repeated below, the static force per equation 14-14, also repeated below, must now be subtracted from the new total force, F_T at velocity v per equation 14-31, to obtain the net magnetic force, F_M.

(14-20) $\quad \mathbf{F_T - F_E = F_M}$

(14-14) $\quad F_E = 2 \cdot R \cdot F_r$

The final step in the calculation is, then, to evaluate equation 14-31 for each of cases 1, 2 and 5 by inserting the values of A, B, C and D from the above Figure 14-12, and then determining F_M from the above two equations. The results are tabulated in Figure 14-13, below.

$$\begin{array}{lll}
\text{Case} & \text{Range} & A \cdot B + C + D \\
1 \rightarrow & <0 & [1+{}^{v}/c]\cdot[1-{}^{v}/c] - {}^{v}/c + {}^{v}/c = 1 - v^2/c^2 \\
\rightarrow & >0 & [1+{}^{v}/c]\cdot[1-{}^{v}/c] - {}^{v}/c + {}^{v}/c = \underline{1 - v^2/c^2}
\end{array}$$

$$\begin{aligned}
\text{Sum} &= 2\cdot(1 - v^2/c^2) \\
\uparrow F_T &= 2\cdot R\cdot F_r(1 - v^2/c^2) \\
\uparrow F_M &= F_T - F_E \\
&= F_T - 2\cdot R\cdot F_r \\
&= -2\cdot R\cdot F_r\cdot(v^2/c^2) \\
&\quad \text{(attraction)}
\end{aligned}$$

$$\begin{array}{lll}
2 \rightarrow & <0 & [1-{}^{v}/c]\cdot[1-{}^{v}/c] + {}^{v}/c + {}^{v}/c = 1 + v^2/c^2 \\
\leftarrow & >0 & [1+{}^{v}/c]\cdot[1+{}^{v}/c] - {}^{v}/c - {}^{v}/c = \underline{1 + v^2/c^2}
\end{array}$$

$$\begin{aligned}
\text{Sum} &= 2\cdot(1 + v^2/c^2) \\
\uparrow F_T &= 2\cdot R\cdot F_r(1 + v^2/c^2) \\
\uparrow F_M &= F_T - F_E \\
&= F_T - 2\cdot R\cdot F_r \\
&= +2\cdot R\cdot F_r\cdot(v^2/c^2) \\
&\quad \text{(repulsion)}
\end{aligned}$$

$$\begin{array}{lll}
5 \rightarrow & <0 & [1-{}^{v}/c] \cdot 1 + {}^{v}/c + 0 = 1 \\
\cdot & >0 & [1+{}^{v}/c] \cdot 1 - {}^{v}/c + 0 = \underline{1}
\end{array}$$

$$\begin{aligned}
\text{Sum} &= 2 \\
\uparrow F_T &= 2\cdot R\cdot F_r \\
\uparrow F_M &= F_T - F_E \\
&= F_T - 2\cdot R\cdot F_r \\
&= 0 \text{ (no effect)}
\end{aligned}$$

Figure 14-13

The above results agree exactly in direction with the force, F_M, for the three cases: 1, 2, and 5 of Figure 14-1. They agree exactly in magnitude with the force, F_M, per the earlier derivation from traditional 20th Century physics that the ratio of the magnetic force to the electrostatic force is $[v^2/c^2]$, equation 14-19. Here, however, the magnetic effect is derived from the characteristics and behavior of centers-of-oscillation. As stated earlier, magnetic field is merely the effect of changes in the electrostatic field effect due to changes in the oscillations of the centers and in their propagated waves caused by the centers being in motion rather than at rest.

CASES 3 AND 4 (FIGURE 14-1)

So far the analysis has been, for cases 1, 2, and 5 of Figure 14-1, of that component, $\uparrow F_T$, that is perpendicular to I_1, vertical in the figures and symbols, of the total force, $\boldsymbol{F_T}$, of interaction between the two parallel currents. The other component, that parallel to I_1, horizontal in the figures, was observed to be a net zero in the static case because of exact mirror symmetry relative to the $x = 0$ point. Now that net zero must be verified for the case of the centers in motion. The same procedure as used above can now be used but can be presented more briefly.

Referring back to equation 14-11, repeated below,

$$(14\text{-}11)\qquad d\vec{F}(x) = \mathbf{dF(x)}\cdot\frac{x}{R(x)} = \mathbf{dF(x)}\cdot\frac{x}{[x^2 + R^2]^{½}}$$

$$\uparrow dF(x) = \mathbf{dF(x)}\cdot\frac{R}{R(x)} = \mathbf{dF(x)}\cdot\frac{R}{[x^2 + R^2]^{½}}$$

the only difference in the expression for calculating the horizontal (in the figures) component of the magnetic field from that for the vertical is that the factor R in the numerator of the expression for $dF(x)$ is replaced with x. Equation 14-12 now becomes as equation 14-32, below, and equation 14-14 becomes as equation 14-33, below.

$$(14\text{-}32)\qquad d\vec{F}(x) = F_r\cdot\frac{R^2\cdot x}{[x^2 + R^2]^{1½}}\cdot dx$$

$$(14\text{-}33)\qquad \vec{F}_E = \int_{-\infty}^{0} d\vec{F}(x) + \int_{0}^{+\infty} d\vec{F}(x)$$

This is the formulation for calculating the static case force on the charge Q_2 in the direction parallel to I_1. The integration and evaluation are at detail notes *DN 6 - Integration Details for Magnetic Field Derivations, Part (3).*

It evaluates to

$$(14\text{-}34)\qquad \vec{F}_E = 0$$

of course, as there is no net such force in the situation configured. The charges of I_1 located to the left of Q_2 exert on Q_2 an electrostatic force component, horizontal in Figure 14-3, repelling Q_2 to the right, which is exactly cancelled by the corresponding force due to the charges of I_1 located to the right of Q_2. (See

Figures 14-2 and 14-3, earlier, above. For the earlier case of the vertical component the corresponding result calculated was $2 \cdot R \cdot F_r$.)

Now, again introducing $f(v,x)$ with the same purpose as before, the expression to be integrated, instead of equation 14-30, is

$$\text{(14-35)} \qquad \vec{F}_T = \int_{-\infty}^{0} dF(\vec{v},x) + \int_{0}^{+\infty} dF(\vec{v},x)$$

$$= \int_{-\infty}^{0} f(\vec{v},x) \cdot dF(x) + \int_{0}^{+\infty} f(\vec{v},x) \cdot dF(x)$$

The integration and evaluation, again are quite complicated, are at detail notes *DN 6 - Integration Details for Magnetic Field Derivations, Part (4)*.

The result is

$$\text{(14-36)} \qquad \vec{F}_T = 0$$

that is, for Cases 1, 2 and 5 of Figure 14-1 there is no interaction force parallel to I_1, as should be the case in agreement with traditional physics.

This somewhat lengthy and involved analysis can be summarized, so far, as follows.

> (1) Of the five Cases of relative motions of two currents, as set out initially in Figure 14-1 and out of which any other situation can be obtained by an appropriate combination, Cases 1, 2 and 5 have been analyzed.
>
> (2) It has been shown in detail for those three cases that the exact magnetic behavior known to exist, and as described by traditional physics, naturally arises from the Coulomb interactions of centers-of-oscillation.

To address the problem of Cases 3 and 4 where I_2 is perpendicular to I_1 and moving toward I_1 in Case 3 and away in Case 4, the same analytic structure can be used with minor changes to account for the different direction of I_2. Referring to Figure 14-11, the analysis depicted was of the effect of I_1 on a single moving charge increment of I_2. The direction in which that charge increment was moving was reflected in the structure of $f(v,x)$ per equation 14-28 and the values of A, B, C and D per Figure 14-12. That $f(v,x)$ was for motion of I_2 parallel to I_1.

Now the charge increment in I_2 is moving perpendicular to I_1, toward it for Case 3 and away from it for Case 4. For those two cases the orientation of center-of-oscillation #2 is now rotated counter-clockwise 90° from its Case 1 and 2, respective orientations in the earlier configuration. As a result the following trigonometric relationships will be needed.

$$\text{(14-37)} \qquad \begin{array}{ll} \mathrm{Sin}(90°-\theta) = \mathrm{Cos}\,\theta & \mathrm{Sin}^2(90°-\theta) = \mathrm{Cos}^2\theta \\ \mathrm{Cos}(90°-\theta) = \mathrm{Sin}\,\theta & \mathrm{Cos}^2(90°-\theta) = \mathrm{Sin}^2\theta \end{array}$$

This makes $f(v,x)$ become equation 14-38, on the following page, for Cases 3 and 4, which equation is analogous to equation 14-28 for Cases 1, 2, and 5.

(14-38) $$f(v,x) = \frac{dF(v,x)}{dF(x)}$$ [Cases 4, 5]

$$= \left[\frac{\text{Wave Resultant in direction } \theta \text{ at velocity v}}{\text{Wave Resultant in direction } \theta \text{ at v=0}}\right] \cdot \left[\frac{\text{Center Resultant in direction } [\mathbf{90}°-\theta] \text{ at v}}{\text{Center Resultant in direction } [\mathbf{90}°-\theta] \text{ at v=0}}\right] + \left[\begin{matrix}F_{fc} \text{ Wve} \\ \text{toward} \\ \theta \\ \text{at v}\end{matrix}\right] - \left[\begin{matrix}F_{fc} \text{ Ctr} \\ \text{toward} \\ \mathbf{90}°-\theta \\ \text{at v}\end{matrix}\right]^{*}$$

* At zero velocity there is no F_{fc} at all.

$$= \frac{\left[\dfrac{\cos^2\theta}{A^2} + \sin^2\theta\right]^{-½}}{1} \cdot \frac{\left[\dfrac{\mathbf{\sin^2}\theta}{B^2} + \mathbf{\cos^2}\theta\right]^{-½}}{1} + \left[C\cdot\cos\theta - D\cdot\mathbf{\sin}\,\theta\right] \quad {**}$$

** Per equation 14-26, *f(v,x)* is a measure of relative effects and does not include F_r.

The Changes in the above equation 14-38 from equation 14-28 are bold in the above and below.

Equations 14-37 have already been applied, and:

A = wave amplitude function of velocity
= $[1-{}^{v}/c]$ forward and $[1+{}^{v}/c]$ rearward

B = center amplitude function of velocity
= $[1-{}^{v}/c]$ forward and $[1+{}^{v}/c]$ rearward

C = wave F_{fc} function of velocity
= $[+{}^{v}/c]$ forward and $[-{}^{v}/c]$ rearward

D = center F_{fc} function of velocity
= $[+{}^{v}/c]$ forward and $[-{}^{v}/c]$ rearward.

Using equation 14-9 for *R(x)* and the right triangle geometry of Figure 14-11, equation 14-38 becomes

(14-39) $f(v,x) =$ [Cases 4, 5]

$$= \left[\frac{x^2/R(x)^2}{A^2} + \frac{R^2}{R(x)^2}\right]^{-½} \cdot \left[\frac{\mathbf{R}^2/R(x)^2}{B^2} + \frac{\mathbf{x}^2}{R(x)^2}\right]^{-½} + \frac{C\cdot x \ \mathbf{- D\cdot R}}{R(x)}$$

$$= \frac{A\cdot B\cdot(x^2 + R^2)}{[B^2\cdot x^4 + (1 + A^2\cdot B^2)\cdot R^2\cdot x^2 + A^2\cdot R^4]^{½}} + \frac{(C\cdot x - D\cdot R)}{(x^2 + R^2)^{½}}$$

Following is a table of the values of A, B, C and D which, however, will not be needed.

Case	Range	A	B	C	D
3 $\rightarrow$	<0	$[1-{}^{V}/c]$	$[1-{}^{V}/c]$	$+{}^{V}/c$	$+{}^{V}/c$
$\uparrow$	>0	$[1+{}^{V}/c]$	$[1-{}^{V}/c]$	$-{}^{V}/c$	$+{}^{V}/c$
4 $\rightarrow$	<0	$[1-{}^{V}/c]$	$[1+{}^{V}/c]$	$+{}^{V}/c$	$-{}^{V}/c$
$\downarrow$	>0	$[1+{}^{V}/c]$	$[1+{}^{V}/c]$	$-{}^{V}/c$	$-{}^{V}/c$

Figure 14-14

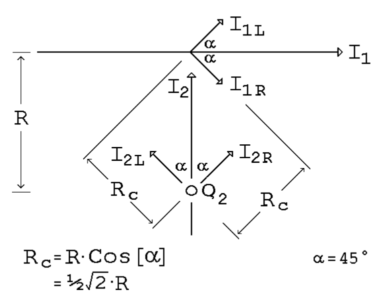
Proceeding to integrate equation 14-35 again, but with the revised $f(v,x)$ of equation 14-38, where the details are given in detail notes *DN 6 - Integration Details for Magnetic Field Derivations, Part (5)*, the result is no force

(14-40) $\quad \vec{F}_T = 0$

that is, two perpendicular currents have no magnetic force resulting from the interaction just investigated. (An examination of the details of the waves encountering the centers-of-oscillation in I_2 in this situation and of the behavior that the centers in I_2 present to the waves shows that there is no substantial difference between the approaching and departing cases. There is no way that the configuration could produce forces in opposite directions for the approaching and departing cases. Thus the zero net interaction is further reasonable for that reason.)

The magnetic effect between perpendicular currents does not come about as a result of the direct perpendicular motion of the two currents. Then it must come about through some other mechanism. That mechanism is that each of the two perpendicular currents has a component that is parallel to a component of the other. To derive the force of interaction between two perpendicular currents from the already-obtained force between parallel currents, which is the actual cause, the procedure is as follows (Figure 14-15).

Step (1) - Each of the two orthogonal currents, I_1 and I_2, are resolved into two components, one to the left and one to the right. Thus, I_1 is



the resultant of I_{1L} and I_{1R} and similarly for I_2.

$I_{1R} = I_{1L} = ½\sqrt{2} \cdot I_1$ $\qquad$ I_{1L} attracts I_{2R}

$I_{2R} = I_{2L} = ½\sqrt{2} \cdot I_2$ $\qquad$ I_{1R} repels I_{2L}

Figure 14-15

Step (2) - I_{2R} is attracted by I_{1L} (Case 1) in the amount

(14-41)
$$F_A = \frac{\mu}{2\pi}\cdot\frac{I_{1L}\cdot I_{2R}}{R_C} \qquad \text{[using equation 14-3]}$$
$$= \frac{\mu}{2\pi}\cdot\frac{\tfrac{1}{2}\sqrt{2}\cdot I_1 \cdot \tfrac{1}{2}\sqrt{2}\cdot I_2}{R_C}$$
$$= \frac{\mu}{2\pi}\cdot\frac{I_1\cdot I_2}{2\cdot R_C}$$

and I_{2L} is repelled by I_{1R} (Case 2) in the amount

(14-42)
$$F_B = \frac{\mu}{2\pi}\cdot\frac{I_{1R}\cdot I_{2L}}{R_C}$$
$$= \frac{\mu}{2\pi}\cdot\frac{\tfrac{1}{2}\sqrt{2}\cdot I_1 \cdot \tfrac{1}{2}\sqrt{2}\cdot I_2}{R_C}$$
$$= \frac{\mu}{2\pi}\cdot\frac{I_1\cdot I_2}{2\cdot R_C}$$

These two forces are depicted in Figure 14-16 below.

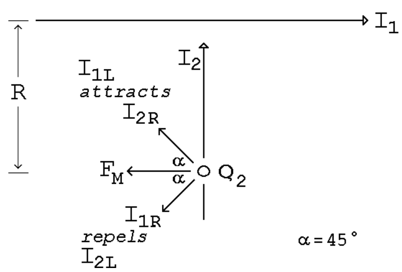


Figure 14-16

Step (3) - The resultant of the two forces, F_A and F_B, is F_m, the net magnetic force, and its magnitude per equation 14-43 and its direction per Figure 14-16 are correct for Case 3.

(14-43)
$$F_M = \sqrt{2}\cdot F_A = \sqrt{2}\cdot F_B$$
$$= \sqrt{2}\cdot\frac{\mu}{2\pi}\cdot\frac{I_1\cdot I_2}{2\cdot R_C}$$
$$= \frac{\mu}{2\pi}\cdot\frac{I_1\cdot I_2}{R} \qquad \text{[From Figure 14-14 } R_C = \tfrac{1}{2}\sqrt{2}\cdot R\text{]}$$

If the direction of I_2 is reversed Case 4 is obtained and a review of the above analysis will show that the resulting magnitude of F_M is the same as for Case 3 while the direction of F_M is opposite to that of Case 3.

Thus the magnetic force in the originally presented Cases 3 and 4 is demonstrated, which completes the derivation of magnetic field from Universal Physics considerations. As with Coulomb's Law and electrostatics in an earlier section, Ampere's Law and magnetostatics are now moved from the realm of empirical results to derived results, results derived from the origin of the universe and its description in this Universal Physics.

(This Section 14 is here interrupted in order to present the details of the several integrations presented in the preceding development.

The section resumes after detail notes *DN 6 - Integration Details for Magnetic Field Derivations*.)