DETAIL NOTES - 6

Integration Details for Magnetic Field Derivations

(See detail notes *DN 1 - Differential Calculus, Derivatives* and detail notes *DN 5 - Integral Calculus* for an explanation of the differential and integral calculus involved.)

PART (1) -- EQUATION 14-14, THE STATIC CASE, ↑ COMPONENT

(14-14)
$$\uparrow F_E = \int_{-\infty}^{0} \uparrow dF(x) + \int_{0}^{+\infty} \uparrow dF(x)$$

$$= \int_{-\infty}^{0} F_r \cdot \frac{R^3}{[x^2 + R^2]^{1\frac{1}{2}}} \cdot dx + \int_{0}^{+\infty} F_r \cdot \frac{R^3}{[x^2 + R^2]^{1\frac{1}{2}}} \cdot dx$$

Since the form of the integral in each of the two regions is the same most of the integration process can be performed on just the form.

(DN6-1)
$$\text{Form} = \int_{a}^{b} F_r \cdot \frac{R^3}{[x^2 + R^2]^{1\frac{1}{2}}} \cdot dx$$

$$= F_r \cdot R^3 \cdot \int_{a}^{b} \frac{1}{[x^2 + R^2]^{1\frac{1}{2}}} \cdot dx$$

[F_r and R are constants relative to the integration.]

$$= F_r \cdot R^3 \cdot \left[\frac{x}{R^2 \cdot [x^2 + R^2]^{\frac{1}{2}}}\right]_a^b$$

[The integration antiderivative]

$$= F_r \cdot R \cdot \left[\frac{1}{[1 + R^2/x^2]^{\frac{1}{2}}}\right]_a^b$$

[Divide numerator and denominator by $R^2 \cdot x$.]

Returning to the overall equation 14-14 and evaluating at the limits:

$$\text{(DN6-2)}\qquad \begin{aligned} \uparrow F_E &= F_r\cdot R\cdot[0 - {}^{\pm}1] && \text{For: } a = -\infty,\ b = 0 \quad [R^2/_{\infty}2 = 0 \text{ and } R^2/_{0}2 = \infty] \\ &= F_r\cdot R\cdot[{}^{\pm}1 - 0] && \text{For: } a = 0,\ b = +\infty \\ &= 2\cdot R\cdot F_r && \text{Overall} \end{aligned}$$

PART (2) -- EQUATION 14-30, CASES 1, 2, & 5, ↑ COMPONENT

$$\text{(14-30)}\qquad \begin{aligned} \uparrow F_T &= \int_{-\infty}^{0} \uparrow dF(v,x) + \int_{0}^{+\infty} \uparrow dF(v,x) \\ &= \int_{-\infty}^{0} \uparrow f(v,x)\cdot dF(x) + \int_{0}^{+\infty} \uparrow f(v,x)\cdot dF(x) \\ &= \int_{-\infty}^{0} \left[\left[\frac{A\cdot B\cdot(x^2 + R^2)}{[x^4 + (A^2 + B^2)\cdot R^2\cdot x^2 + A^2\cdot B^2\cdot R^4]^{½}} + \ldots\right.\right. \\ &\qquad \ldots + \left.\frac{(C + D)\cdot x}{(x^2 + R^2)^{½}}\right]\cdot\left.\left[F_r\cdot\frac{R^3}{[x^2 + R^2]^{1½}}\right]\right]\cdot dx + \ldots \\ &\qquad \ldots + \int_{0}^{-\infty}\left[\begin{matrix}\text{The same above entire}\\ \text{expression a second}\\ \text{time}\end{matrix}\right]\cdot dx \end{aligned}$$

Again, since the form of the two integrals in equation 14-30 is the same, only one need be followed through the integration process up to the point of inserting the limits to evaluate the integral. The integral to be dealt with is

$$\text{(DN6-3)}\qquad \begin{aligned} \text{Form} &= \int_{a}^{b} f(v,x)\cdot dF(x) \qquad \text{[and, integrating by parts, becomes]} \\ &= \left[f(v,x)\cdot F(x) - \int F(x)\cdot df(v,x)\right]_{a}^{b} \end{aligned}$$

where, from equation 14-30

$$f(v,x) = \left[\frac{A\cdot B\cdot(x^2 + R^2)}{[x^4 + (A^2 + B^2)\cdot R^2\cdot x^2 + A^2\cdot B^2\cdot R^4]^{½}} + \ldots\right.$$

$$\ldots + \left.\frac{(C + D)\cdot x}{(x^2 + R^2)^{½}}\right]$$

and, from the integration of equation 14-14 at the next to last line of equation DN6-1

$$F(x) = F_r\cdot R^3\cdot\left[\frac{x}{R^2\cdot[x^2 + R^2]^{½}}\right]$$

The first of these two terms is a simple algebraic product of two functions and can readily be evaluated. The second term, that having the integral, evaluates to zero as shown in the following and can therefore be ignored.

TO SHOW THAT THE SECOND TERM IS ZERO

Step 1

The "order" (the highest exponent magnitude) of the derivative of a polynomial expression is one less than the order of the polynomial expression.

(DN6-4)

(1) Define Ord[...] = the order of [...]

(2) $\mathrm{Ord}\left[(ax^n + bx^{n-1} + \ldots)^m\right] = m \cdot n$

(3) $\frac{d}{dx}\left[ax^n + bx^{n-1} + \ldots\right]^m = \ldots$

$$= m \cdot \left[ax^n + bx^{n-1} + \ldots\right]^{m-1} \cdot \left[n \cdot ax^{n-1} + (n-1)bx^{n-2} + \ldots\right]$$

(4) $\mathrm{Ord}\left[(3),\ \text{above}\right] = (m-1) \cdot n + (n-1)$

$$= m \cdot n - 1$$

$$= \mathrm{Ord}[(2),\ \text{above}] - 1$$

Step 2

Given two polynomial expressions, $u(x)$ and $v(x)$, then

(DN6-5) $$\mathrm{Ord}\left[\frac{u(x)}{v(x)}\right] = \mathrm{Ord}\left[\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right]\right] - 1$$

(1) Let $\mathrm{Ord}\left[u(x)\right] = U$ and $\mathrm{Ord}\left[v(x)\right] = V$

(2) $\mathrm{Ord}\left[\frac{u(x)}{v(x)}\right] = U - V$

(3) $d\left[\frac{u(x)}{v(x)}\right] = \frac{v \cdot d[u(x)] - u \cdot [d(v(x))]}{v(x)^2}$

(4) $\mathrm{Ord}\left[d\left[\frac{u(x)}{v(x)}\right]\right] = \left[[V+(U-1)] \text{ or } [U+(V-1)]\right] - 2 \cdot V$

$$= [U + V - 1] - 2 \cdot V = U - V - 1$$

$$= \mathrm{Ord}\left[\frac{u(x)}{v(x)}\right] - 1$$

In words, given the quotient of two polynomial expressions in the same variable, the process of differentiation increases the order of the quotient by 1.

The process of integration is anti-differentiation; the integral of a function, $f(x)$ is another function $g(x)$ such that the derivative of $g(x)$

equals $f(x)$. Therefore, since differentiation increases the order of the quotient by one then the process of integration reduces the order of the quotient by 1.

(DN6-6)
$$\mathrm{Ord}\left[\int \frac{u(x)}{v(x)} \cdot dx\right] = \mathrm{Ord}\left[\frac{u(x)}{v(x)}\right] - 1$$

Step 3

Now, proceeding with the part of the expression that is said to be equal to zero, namely the second term of the integration by parts of equation DN6-3:

(DN6-7)
$$\mathrm{Form} = \Big[f(v,x) \cdot F(x) - \underbrace{\int F(x) \cdot df(v,x)\Big]_a^b}_{\text{2nd term}}$$

(1) $f(v,x)$ = per equation DN6-3, above.

(2) $\mathrm{Ord}[df(v,x)] = \mathrm{Ord}[f(v,x)] - 1$
$= (2 - 2) - 1 = -1$

(3) $F(x)$ = per equation DN6-3, above.

(4) $\mathrm{Ord}[F(x)] = 1 - 1 = 0$

(5) $\mathrm{Ord}\left[F(x) \cdot df(v,x)\right] = 0 + (-1) = -1$

Since the order of the integrand (the expression that is to be integrated) is -1, the order of the resulting integral is -2, per the above theorems.

When the integral is evaluated at the limit $+\infty$ or $-\infty$ the result will be zero. The integral may evaluate to some finite amount when evaluated at the other limit, 0, but that finite amount will be the same for evaluating the first term of equation 14-30 (evaluation range $-\infty$ to 0) as for evaluating the second term (evaluation range 0 to $+\infty$). Since the 0 is the upper limit in the one case and the lower limit in the other its effect cancels in the overall integration from $-\infty$ to $+\infty$. Thus the overall second term integrates and evaluates to zero.

The integration and evaluation of equation 14-30 now has reduced to just the first term, an algebraic expression without any integration involved,

(DN6-8)
$$\uparrow F_T = \Big[f(v,x) \cdot F(x)\Big]_{-\infty}^{0} + \Big[f(v,x) \cdot F(x)\Big]_{0}^{+\infty}$$

where

$$f(v,x) = \frac{A \cdot B \cdot (x^2 + R^2)}{[x^4 + (A^2 + B^2) \cdot R^2 \cdot x^2 + A^2 \cdot B^2 \cdot R^4]^{1/2}} + \frac{(C + D) \cdot x}{(x^2 + R^2)^{1/2}}$$

and

$$F(x) = F_r \cdot R \cdot \left[\frac{x}{[x^2 + R^2]^{1/2}}\right]$$

Dividing the numerator and the denominator of the first term of $f(v,x)$ by x^2 and of the second term by x, it is apparent that the value of the function for $x = \pm\infty$ is $\pm[A \cdot B + C + D]$. Similarly, $F(x)$ evaluates to ± 1 for $x = \pm\infty$. $F(x)$ evaluates to zero for $x = 0$. Therefore the final result is

(DN6-9)
$$\uparrow F_T = R \cdot F_r \cdot \left[A \cdot B + C + D\right]$$

PART (3) -- EQUATIONS 14-32 & 33, THE STATIC CASE, → COMPONENT

(14-32)
$$d\vec{F}(x) = F_r \cdot \frac{R^2 \cdot x}{[x^2 + R^2]^{1½}} \cdot dx$$

(14-33)
$$\vec{F_E} = \int_{-\infty}^{0} d\vec{F}(x) + \int_{0}^{+\infty} d\vec{F}(x)$$

Since the form of the integral in each of the two regions is the same, most of the integration process can be performed on just the form.

(DN6-10)
$$\text{Form} = \int_a^b F_r \cdot \frac{R^2 \cdot x}{[x^2 + R^2]^{1½}} \cdot dx$$

$$= F_r \cdot R^2 \cdot \int_a^b \frac{x}{[x^2 + R^2]^{1½}} \cdot dx$$ [F_r and R are constants relative to the integration.]

$$= F_r \cdot R^2 \cdot \left[\frac{-1}{[x^2 + R^2]^{½}}\right]_a^b$$ [The integration antiderivative]

Returning to the overall equation and evaluating at the limits:

(DN6-11)
$$\vec{F_E} = -F_r \cdot R^2 \cdot \left[\pm\frac{1}{R} - 0\right]$$ For: $a = -\infty$, $b = 0$ [$R^2/_{\infty}2 = 0$ and $R^2/_{0}2 = \infty$]

$$= -F_r \cdot R^2 \cdot \left[0 - \pm\frac{1}{R}\right]$$ For: $a = 0$, $b = +\infty$

$$= 0$$ Overall

PART (4) -- EQUATION 14-35, CASES 1, 2 & 5, → COMPONENT

(14-35)
$$\vec{F_T} = \int_{-\infty}^{0} f(\vec{v},x) \cdot dF(x) + \int_{0}^{+\infty} f(\vec{v},x) \cdot dF(x)$$

Again, since the form of the two integrals in 14-35 is the same, only one of them need be followed through the integration process up to inserting the limits to evaluate the integral. The integral to be dealt with is then

(DN6-12)
$$\text{Form} = \int_a^b f(v,x)\cdot dF(x) \qquad \text{[and, integrating by parts, becomes]}$$

$$= \left[f(v,x)\cdot F(x) - \int F(x)\cdot df(v,x) \right]_a^b$$

where $f(v,x)$ is the same as before, from equation 14-30, and F(x), from the integration of equation 14-33 at the last line of equation DN6-10, is

$$F(x) = F_r\cdot R^2\cdot\left[\frac{-1}{[x^2 + R^2]^{½}}\right]$$

The first of these two terms is a simple algebraic product of two functions and can readily be evaluated. The second term, that having the integral again evaluates, to zero as shown in the following and, therefore, can be ignored.

TO SHOW THAT THE SECOND TERM IS ZERO

(The reasoning starts with the third step of the analogous reasoning in Part (2), above, since the first two steps are the proof of general theorems applicable to both steps.)

Step 3: Proceeding with the part of the expression that is said to be equal to zero, namely:

(DN6-13)
$$\text{Form} = \left[f(v,x)\cdot F(x) - \underbrace{\int F(x)\cdot df(v,x) \Big]_a^b}_{\text{2nd term}} \right.$$

(1) $f(v,x)$ = per equation DN6-3, above.

(2) Ord[d$f(v,x)$] = Ord[$f(v,x)$] - 1

= (2 - 2) - 1 = -1

(3) F(x) = per equation DN6-12, above.

(4) Ord[F(x)] = 0 - 1 = -1

(5) Ord $\left[F(x)\cdot df(v,x)\right]$ = -1 + (-1) = -2

Since the order of the integrand (the expression that is to be integrated) is *-2*, the order of the resulting integral is *-3*, per the theorems.

When the integral is evaluated at the limit $+\infty$ or $-\infty$ the result will be zero. The integral may evaluate to some finite amount when evaluated at the other limit, *0*, but that finite amount will be the same for evaluating the first term of equation 14-35 (evaluation range $-\infty$ to 0) as for evaluating the second term (evaluation range 0 to $+\infty$). Since the 0 is the upper limit in the one case and the lower limit in the other its effect cancels in the overall integration from $-\infty$ to $+\infty$. Thus the overall second term integrates and evaluates to zero.

The integration and evaluation of 14-35 now has reduced to just the first term, an algebraic expression without any integration involved,

(DN6-14)
$$\vec{F}_T = \Big[f(v,x)\cdot F(x)\Big]_{-\infty}^{0} + \Big[f(v,x)\cdot F(x)\Big]_{0}^{+\infty}$$

where

$$f(v,x) = \frac{A\cdot B\cdot(x^2 + R^2)}{[x^4 + (A^2 + B^2)\cdot R^2\cdot x^2 + A^2\cdot B^2\cdot R^4]^{½}} + \frac{(C + D)\cdot x}{(x^2 + R^2)^{½}}$$

and

$$F(x) = F_r\cdot R^2\cdot\left[\frac{-1}{[x^2 + R^2]^{½}}\right]$$

If the numerator and the denominator are each divided by x^2 in the first term of $f(v,x)$ and by x in the second term, it is apparent that for $x = \pm\infty$ the value of $f(v,x)$ is $\pm[A\cdot B + C + D]$. Similarly, $F(x)$ evaluates to 0 for $x = \pm\infty$. The product of the two, the overall value of the function is thus 0 at $\pm\infty$. For $x = 0$ the value of $f(v,x)$ is 1 and $F(x)$ evaluates to zero. The product of those is zero, also. Therefore the final result is zero overall.

(DN6-15) $\vec{F}_T = 0$

PART (5) -- EQUATION 14-35, AGAIN, BUT WITH STRUCTURE FOR CASES 3 AND 4, → COMPONENT

(14-35)
$$\vec{F}_T = \int_{-\infty}^{0} f(\vec{v},x)\cdot dF(x) + \int_{0}^{+\infty} f(\vec{v},x)\cdot dF(x)$$

Once more, since the form of the two integrals in 14-35 is the same, only one of them need be followed through the integration process up to the point of inserting the limits to evaluate the integral. The integral to be dealt with is then

(DN6-16)
$$\text{Form} = \int_a^b f(v,x)\cdot dF(x) \qquad \text{[and, integrating by parts, becomes]}$$

$$= \Big[f(v,x)\cdot F(x) - \int F(x)\cdot df(v,x)\Big]_a^b$$

where F(x), again from the integration of equation 14-33 at the last line of equation DN6-10, is

$$F(x) = F_r\cdot R^2\cdot\left[\frac{-1}{[x^2 + R^2]^{½}}\right]$$

(DN6-16, continued)

and $f(v,x)$ is the new form for Cases 3 and 4 from equation 14-39

$$f(v,x) = \frac{A \cdot B \cdot (x^2 + R^2)}{[B^2 \cdot x^4 + (1 + A^2 \cdot B^2) \cdot R^2 \cdot x^2 + A^2 \cdot R^4]^{1/2}} + \ldots$$

$$\ldots + \frac{(C \cdot x - D \cdot R)}{(x^2 + R^2)^{1/2}}$$

The first of these two terms is a simple algebraic product of two functions and can readily be evaluated. The second term, that having the integral again evaluates, to zero as shown in the following and, therefore, can be ignored.

TO SHOW THAT THE SECOND TERM IS ZERO

(The reasoning starts with the third step of the analogous reasoning in Part (2), above, since the first two steps are the proof of general theorems applicable to both steps.)

Step 3: Proceeding with the part of the expression that is said to be equal to zero, namely:

(DN6-17)

$$\text{Form} = \Big[f(v,x) \cdot F(x) - \underbrace{\int F(x) \cdot d f(v,x)}_{\text{2nd term}} \Big]_a^b$$

(1) $f(v,x)$ = per equation DN6-16, above.

(2) Ord[d$f(v,x)$] = Ord[$f(v,x)$] - 1
= (2 - 2) - 1 = -1

(3) F(x) = per equation DN6-16, above.

(4) Ord[F(x)] = 0 - 1 = -1

(5) Ord $\left[F(x) \cdot d f(v,x)\right] = -1 + (-1) = -2$

Since the order of the integrand (the expression that is to be integrated) is *-2*, the order of the resulting integral is *-3*, per the earlier theorems.

When the integral is evaluated at the limit $+\infty$ or $-\infty$ the result will be zero. The integral may evaluate to some finite amount when evaluated at the other limit, *0*, but that finite amount will be the same for evaluating the first term of equation 14-35 (evaluation range $-\infty$ to *0*) as for evaluating the second term (evaluation range *0* to $+\infty$). Since the *0* is the upper limit in the one case and the lower limit in the other its effect cancels in the overall integration from $+\infty$ to $-\infty$. Thus the overall second term integrates and evaluates to zero.

The integration and evaluation of 14-35 now has reduced to just the first term, an algebraic expression without any integration involved, (DN6-18) $$\vec{F}_T = \Big[f(v,x)\cdot F(x)\Big]_{-\infty}^{0} + \Big[f(v,x)\cdot F(x)\Big]_{0}^{+\infty}$$

where $f(v,x)$ and $F(x)$ are per equation DN6-16, above.

If the numerator and the denominator are each divided by x^2 in the first term of $f(v,x)$ and by x in the second term, it is apparent that for $x = \pm\infty$ the value of $f(v,x)$ is $\pm[A\cdot B + C + D]$. Similarly, $F(x)$ evaluates to 0 for $x = \pm\infty$. The product of the two, the overall value of the function is thus 0 at $\pm\infty$. For $x = 0$ the value of $f(v,x)$ is 1 and $F(x)$ evaluates to zero. The product of those is zero, also. Therefore the final result is zero overall.

(DN6-19) $$\vec{F}_T = 0$$

PART (6) -- CASES 3 AND 4, ↑ COMPONENT

In Cses 3 and 4 the two currents are at right angles to each other. Since the action of one current on the other is exactly equal in magnitude and opposite in direction to the action of the other current on the one, the proof in Part (5) above that there is no net → force when the currents are at right angles also proves that there is no net force ↑ in this part (6).

(Section 14 here resumes after the preceding interruption in order to present the magnetic field development integration details.)

ELECTROMAGNETIC FIELD

So far the discussion has treated charges / centers-of-oscillation at rest, which give rise to electrostatic field, and in motion at constant speed, which, in addition, give rise to magnetostatic field. The development has demonstrated the complete compatibility of this Universal Physics with the classical physics of Newton, Coulomb and Ampere, and the related phenomena and laws.

Those phenomena and laws have now passed from being mere empirical deductions by observation of nature to derived results, derived from the origin of the universe and the consequent effects. Of course there was an extremely high level of confidence in Newton's, Coulomb's and Ampere's laws in any case. Consequently, their demonstration from these Universal Physics first principles constitutes a quite strong validation of this Universal Physics and its concepts.

Now it is time to consider other motions, namely those at non-constant speed, that is with change of speed or both speed and direction.

Electromagnetic field results directly from the same considerations and the application of Coulomb's and Ampere's results as developed by Maxwell. One could leave the analysis at that. This Universal Physics being compatible with and functionally identical to the starting point from which Maxwell obtained his development of the theory of electromagnetic phenomena, there is no need to further validate the point. However, by applying these Universal Physics concepts to electromagnetic theory some new results and insights to the actual events and processes are obtained.

Traditional physics treats the phenomena in terms of fields and charges whereas this Universal Physics is in terms of waves and centers-of-oscillation. The discussion must begin in the one set of terms and then proceed to the other in order to relate the two.

Electromagnetic theory is codified in a set of four laws, Maxwell's Equations, which in words are:

(1) Electric field is a consequence of the presence of electric charge or a changing magnetic field

(2) Magnetic field is a consequence of electric current flow or a changing electric field.

(3) The electric field shape is such that it diverges outward from whatever causing charge is present and otherwise closes upon itself.

(4) The magnetic field shape closes upon itself.

These relationships in words are imprecise, of course. The actual laws are in the form of vector differential calculus and are precise with regard to quantities, location and direction. (See detail notes *DN 7 - Maxwell's Equations*). The significance of them here is that they state that changing electric field causes magnetic field (changing magnetic field) and changing magnetic field causes electric field (changing electric field). Consequently the two can continuously produce each other, the appropriate energy being available. The result is the propagation in space of an oscillating electromagnetic field.

In terms of this Universal Physics the development so far has established the nature of static electric and magnetic field. Static electric field is the effects that result from the propagation of U-waves by a center-of-oscillation at rest. Static magnetic field is the effects that result from the changes in the pattern of the U-waves propagated from a center if it is in motion at constant speed rather than being at rest.

The question to be addressed now is, "What are dynamic (non-static) electric field and magnetic field in terms of U-waves ?"

> Since static *electric field* is the U-wave propagation pattern characteristic of stationary centers and is spherically symmetrical relative to the center, then dynamic (changing) electric field is a spherically symmetrical U-wave pattern of propagation from centers whose distribution in space is changing. That is, if the amount of charge, the number of centers, changes then the electric field changes
>
> Since static *magnetic field* is the changes in the pattern of U-wave propagation of a center to other than spherically symmetrical due to motion of the center at constant speed, then dynamic (changing) magnetic field is the result of changes in the motion of the center, changes in its speed.

Of course, it is obvious that

- the distribution of centers' locations in space cannot change except by non-constant speed motion of the centers, and
- if centers are in motion at other than constant speed the motion causes changes in the spatial distribution of the centers.

Thus changing electric field always involves associated changing magnetic field and vice-versa. That is an inevitable consequence of their both being an effect of U-wave propagation from centers. It is the underlying reason for traditional physics' laws that changing electric field induces changing magnetic field and vice versa as expressed in Maxwell's Equations. That changing electric field induces changing magnetic field in free space has not been specifically observed or proven. It was an assumption that Maxwell made, justified at first by arguments of symmetry and reciprocity, and validated by the success of the resulting theory.)

But the terminology "induces" and the thinking associated with it is incorrect. Neither field "induces" or causes the other. Rather, each is merely an aspect of the overall general changing pattern of U-wave propagation when the propagating center(s) have changing motion. The one reality is the pattern of the U-waves. Science has (implicitly) adopted the convention of interpreting some of the aspects of that pattern as what it calls electric field and other aspects as what it calls magnetic field.

E-M (electromagnetic) field is not a dynamic interactive process as it propagates, as Maxwell's Equations are usually interpreted to mean. E-M field is generated, caused, by the motions of charges and then travels outward as an imprint on the U-waves emanating from those charges. The only cause or "inducing" action is the motion of the centers and the changing distribution of them in space. The correct interpretation of Maxwell's Equations is that changing electric and magnetic field are always each associated with the presence of the other.

While in the effects that it produces the E-M field appears to have separate existence, it really does not. It is merely the pattern of modulation of the otherwise uniform and symmetrical pattern of U-waves from static charges. (In a loose sense it is somewhat like amplitude modulation (electric field) and frequency modulation (magnetic field) of a transmitted carrier wave (the static case U-waves) as in AM and FM radio and television signals.) Its existence is like that of waves on the sea. The waves are "shape" but the only substance is sea water.

It is not the E-M waves that propagate in space; it is only the U-waves, which, in traveling through space carry the variations representing E-M field with them. In a crude two dimensional analogy one can imagine a roll of paper with the end taken in someone's hand. The person then runs off pulling the paper and unrolling it from the roll. If beforehand someone had drawn an oscillatory pattern along the length of the paper, then as the roll unwinds and the paper moves out (propagates) the oscillatory pattern on the paper appears to move through space (or also propagate). It travels, of course, at the speed of the paper. The E-M field travels through space at the speed of the U-waves that carry it, that which we call the speed of light, c.

E-M field can result from non-constant motion of either $+U$ or $-U$ centers, positive or negative charges. The U-waves are then either $+U$ or $-U$ waves. But there is only one kind of E-M field. The E-M field is only changes in the U-wave pattern and the changes are of the same nature in $+U$ as in $-U$. The only difference is that, everything else being the same, $+U$ E-M field is $180°$ out of phase with $-U$ E-M field and vice versa. Thus if a $+U$ center and a $-U$ center were hypothetically co-located and had the same varying motion, each would propagate an E-M wave but the waves would be equal in magnitude and opposite in phase, thus canceling each other.

In general, the generation and propagation of E-M waves is called radiation. It appears in an apparent variety of forms, but all are the same in cause and behavior. (There are other forms of radiation which are the actual motion of an energetic particle, not an E-M wave. These are discussed in section *18 - A Model for the Universe (8) - Radioactivity*.)

The various apparent forms differ only in frequency and wavelength. The following Figure 14-17 is a table describing these various different effects. The frequency range divisions in the table are arbitrary and according to custom.

For comparison, the frequency and wavelength of the U-waves of the electron and proton centers-of-oscillation are:

$$f_e = 1.24 \cdot 10^{20} \text{ Hz} \qquad \lambda_e = 2.41 \cdot 10^{-12} \text{ m}$$

$$f_p = 2.27 \cdot 10^{23} \text{ Hz} \qquad \lambda_p = 1.32 \cdot 10^{-15} \text{ m.}$$

Usual natural case of the centers that cause the radiation	Way in which the radiation appears to us	Range Of Freq'y Hz (cps)	Range Of Wavelength as 3·( )m
"Free" electrons in a metal conductor	Radio waves	10^{4}-10^{12}	10^{4}-10^{-4}
Molecules	Heat as infra-red radiation	10^{12}-10^{14}	10^{-4}-10^{-6}
Atomic orbital electrons	Light	upper 10^{14} range	10^{-6} range
Atomic electrons & particles with energy to be free of atomic binding	Ultra-violet light	10^{15}-10^{19}	10^{-7}-10^{-11}
Very high energy particles	X-rays	10^{17}-10^{19}	10^{-9}-10^{-11}
	Gamma rays & Cosmic rays	10^{19}...	10^{-11}...

Figure 14-17

An E-M wave tends to cause an encountered charge to move in the same fashion, in the same direction, and at the same rate as was the motion of the charges whose motion generated the E-M wave. Figures 14-18 and 14-19, below, depict the effect of the electromagnetic wave propagated by an oscillating charge on an encountered charge, treating the magnetic field aspect alone and the electric field aspect alone, respectively.

The Effect of the M-Field part of the E-M Wave
On an Encountered Charge / Center

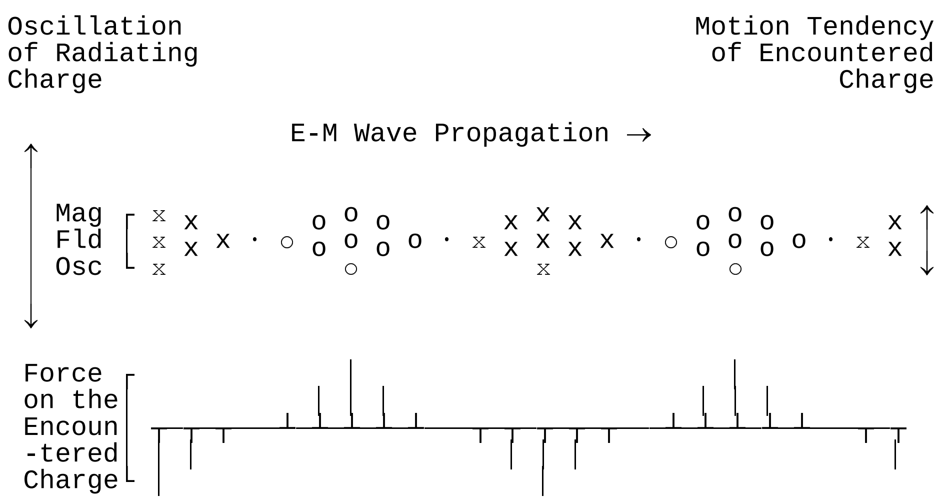

Figure 14-18

The Effect of the E-Field part of the E-M Wave
On an Encountered Charge / Center

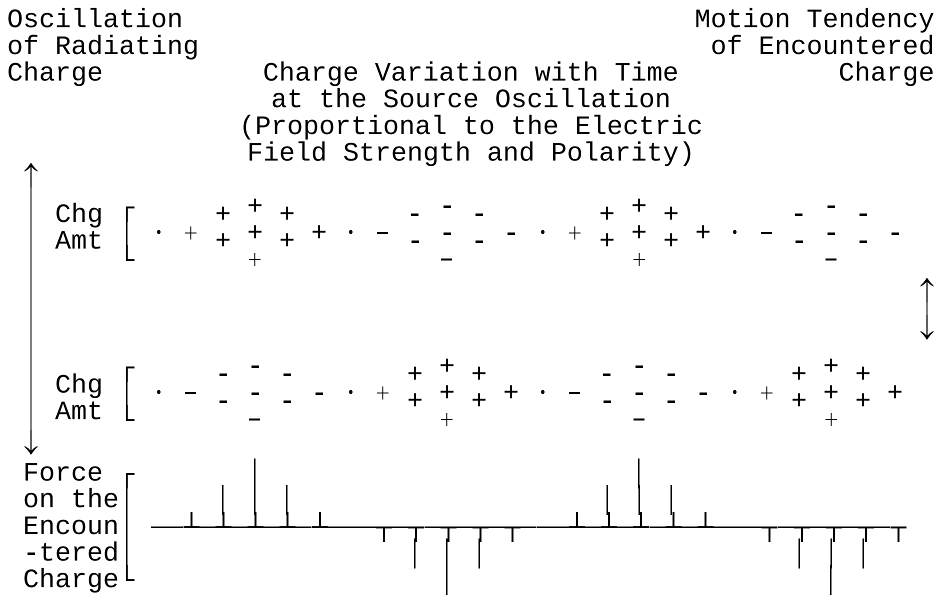

Figure 14-19

The discussion is in terms of sinusoidal oscillations: (a) for the motion variations of the charges / centers-of-oscillation that cause the E-M radiation, (b) consequently for the E-M radiation itself, and (c) as the motion tended to be produced in encountered charges / centers-of-oscillation. This is appropriate for two reasons. First, the Fourier Transform principle shows that any other form of periodic variation is resolvable into a sum of sinusoidal ones, so that the sinusoidal variation is the archetypical. Second, the actual motions of the charges involved in radiation in nature are generally sinusoidal.

The radiation from an individual oscillating charge is polarized, that is it is aligned relative to the motion of the individual radiating charge. The macroscopic radiation due to a large number of radiating charges whose motions are randomly oriented relative to each other will contain a large number of randomly oriented polarizations, which amounts to no polarization at all. That is the case with most cases of natural radiation because of the large number of radiators involved. The radiation from most antennas, on the other hand, tends to be polarized relative to the antenna.

The U-waves, the only actual reality propagating, are longitudinal waves; their variation is in the direction of their propagation. But the E-M field consists of transverse waves, that is the variations in the E-field and the M-field are at right angles to the direction of propagation.

An E-M wave propagates, transmits, energy. This statement calls for a clarification of what energy is. Macroscopically and traditionally energy is defined as "the ability to do work". That definition is not satisfactory at the center / wave level of consideration. Rather, energy must be defined there in terms of the centers' oscillations and the propagated waves since those are the only realities present. If there is energy the "ability to do work" must somehow be in them.

When a center-of-oscillation is at rest its oscillation and its propagation are the same in all directions. It is spherically symmetrical. For a center the spherically symmetrical component of its oscillation is rest energy (energy in rest form).

When a center is in motion its oscillation and its propagation become non-symmetrical in the direction of the motion. It then has what is classically termed "kinetic energy", but it was found in the prior section that in the behavior of centers it is a more correct point of view to address the "energy in kinetic form". The center's deviations from spherical symmetry in its oscillation and its propagation are its energy in kinetic form. Figure 14-20, below, depicts the distinction between those two states and the corresponding shape of the center's oscillation and propagation.

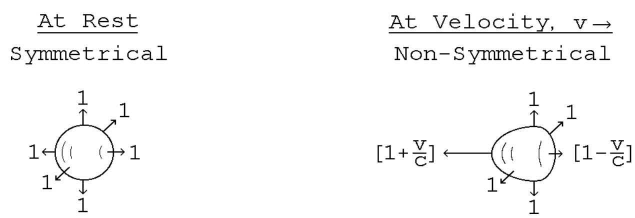

Figure 14-20

Both the rest energy and the energy in kinetic form can be thought of as potential energy when they are constant with time since the constancy means that no changes are taking place, that there is no interaction or energy exchange. Work is done when they vary for any reason, the only possible reason being a change in speed of the center, which change can only result from interaction of the center with other centers' propagated waves.

This last statement is very meaningful, fundamental and important. It can be restated as:

> "Throughout the entire universe *everything that happens is fundamentally changes in the velocities of centers and their propagated waves and such changes only happen by virtue of interactions among centers and their waves*."

All mechanics, optics, thermodynamics, chemistry, biology, etc. are reducible to this. If one takes the general mode of thinking that comprehends the behavior of a gas in terms of the Kinetic Theory of Gases, which treats the gas in terms of the motions of its component molecules, and extends the mode of thinking beneath the atomic level to the charged particles of which atoms are composed, then one has the reality of physics.

For the propagated wave the spherically symmetrical part of the propagated U-wave field (symmetrical relative to each wave's source center) is energy as electric field. The deviations from spherical symmetry are energy as magnetic field. (In the case of a non-spherically shaped group of centers the overall pattern resulting from the sum of the spherical components from each center would have a non-spherical shape like that of the source group of centers, but would nevertheless be energy as electric field because it is the sum of individual propagating centers' spherical components.)

This energy as electric or magnetic field is potential energy if the source center is at rest or at constant speed, because the field is not changing. Each successive cycle of U-wave has the same amplitude, frequency, wavelength and medium density at any point in any specific direction as did the prior cycle in that specific direction.

When the motion of the center-of-oscillation is not at constant speed a variation appears between successive cycles of U-wave propagated from it. The frequency, wavelength and medium density of successive waves varies according to the speed variation of the center and the amplitude of successive waves varies according to the changes in spatial distribution of charge. This non-uniformity of successive U-wave cycles is propagated energy, energy propagated as E-M wave. This point, also, is fundamental and quite important to grasp:

> *"A constant repetition of identical U-waves is merely a potential energy field but changes in the propagated U-waves from cycle to cycle are propagated energy, energy traveling away from the source center."*

(There is more to be presented with regard to this point, particularly on the subject of photons. It is dealt with in the following section *15 - A Model for the Universe (5) - Quanta and the Atom*.)

The U-wave field of a center-of-oscillation at rest, which field extends outward in all directions from the center, is an electric field of potential energy. Energy resides in the field, statically. It is not going anywhere even though the waves which cause it are flowing outward. If the amount of charge of that center were to increase (hypothetically) then the wave amplitude would increase. The new, higher amplitude waves would propagate out at *c*, their velocity of propagation. The new, resulting, electric field would be storing more potential energy than before. That additional energy had to come from the center, there being no other source, and must reside statically in the static E-field as did the original energy there.

If, now the amount of charge were to decrease then the wave amplitude would decrease. The new, lower amplitude would propagate outward at *c*. The now still newer electric field would be storing less energy than before. What happened to the extra energy that was statically stored in the old, larger E-field ? Just as the old, larger amplitude waves continue propagating outward and are followed by the new smaller amplitude ones, so the larger potential energy that was stored in the E-field is replaced with the newer, smaller potential energy.

In other words, if the amount of charge changes then the static electric field's potential energy changes. Increases are energy transmission from the center to the field. Decreases are transmission of the excess energy outward, propagation of energy.

Exactly analogous behavior occurs with regard to the static magnetic field and the effect of changes in the source center's speed. If the action causing E-M wave propagation, non-constant speed motion of the centers-of-oscillation, occurs then potential energy is "pumped" from the center into the "static" E-field and M-field and then propagates out with then as it is replaced by the following waves of newer different "static" energy in the pair of fields.

Such changes occurring regularly, alternately increasing and decreasing, that is oscillatory changes in the spatial distribution of charges and in their velocity, results in E-M radiation and its propagation of E-M energy.

THE DIELECTRIC AND THE SPEED OF LIGHT

The dielectric constant, ε, appears in Coulomb's Law where it affects the magnitude of the electrostatic effect. The permeability, μ, appears in Ampere's Law where it affects the magnitude of the magnetic effect. Through the dependence of E-M waves on those two laws, ε and μ together determine the speed of light, c, because light is a form of E-M wave.

$$c = 1/\sqrt{\mu \cdot \varepsilon} \qquad (14\text{-}44)$$

The reason for this is presented in section 16 where the speed of propagation of U-waves is developed. Since E-M waves are merely an "imprint" on U-waves, the speed of U-waves is the speed of light.

In "free space", by which is meant a region empty of matter and its associated effects, the values of ε and μ are termed ε_0 and μ_0. The resulting value of c is the "natural" or free space value. When not in "free space" the speed of light is slower, that is the values of ε and μ are (or at least their product is) greater than in free space.

This behavior is an aspect of a general characteristic of U-wave propagation that the speed of the waves is slowed by their passing through other U-waves. This behavior is fully developed and analyzed in section 16.

The parameter ε takes account of the effect of a material substance in the region of the electric field. It is usually treated in terms of a relative dielectric constant, k where

$$k = \varepsilon/\varepsilon_0 \qquad (14\text{-}45)$$

In free space, that is with no intervening material substance, $k=1$. But with various substances in the field region k has various greater values. Similarly, the parameter μ takes account of the effect of a material substance in the region of magnetic field.

All of the various E-M wave effects have been described here in general terms. The purpose has been only to indicate how they relate to the wave-center basis of matter and field. The treatment of all of these matters is well developed in traditional 20th Century physics and the Universal Physics basis for the starting point of that development has been established in this and the preceding sections. There is no point to developing all of the treatment over again in specific Universal Physics terms, and, for most practical uses the methods of traditional 20th Century physics are probably less cumbersome (and certainly more familiar).

But there is an important aspect of E-M radiation yet to be dealt with, its "particle" aspect, the photon. That and the related quantum theory and quantum mechanics are addressed in the next section.

Maxwell's Equations

In order to present Maxwell's Equations a brief elaboration of differential calculus, as presented in detail notes *DN 1 - Differential Calculus*, is necessary.

The rate of change of a function with respect to change in its independent variable (for example the rate of change of $U(t)$ with respect to change in t) is called the *derivative of U with respect to t*. It is symbolized

$$\text{(DN7-1)} \qquad \frac{dU(t)}{dt}$$

as presented in detail notes *DN 1*. If the function, U, is a function of more than one variable, the rate of change must be specified with respect to change in each of the variables independently. For example if the function is $U(s,t)$, a function of the two variables, s and t, then one must first treat t as a constant and obtain the derivative with respect only to changes in s then treat s as a constant and obtain the derivative with respect only to changes in t.

Such derivatives are called *partial derivatives* and, to avoid confusing them with the simple derivatives previously dealt with, they are symbolized differently, using ∂ instead of d. Thus the first, second and third partial derivatives of $U(s,t)$ with respect to s are

$$\text{(DN7-2)} \qquad \frac{\partial U(s,t)}{\partial s} \qquad \frac{\partial U^2(s,t)}{\partial s^2} \qquad \frac{\partial U^3(s,t)}{\partial s^3}$$

and with respect to t are analogous. Usually, when the variables involved are clearly understood, they are not written in these expressions. Thus $U(s,t)$ is written simply as U.

Now, to describe time-varying fields in three dimensional space requires four variables, one for each dimension and one for time. The fields are vector quantities (having both magnitude and direction) and must be resolved into one dimensional components to be treated mathematically. For example, the electric field is

$$\text{(DN7-3)} \qquad \vec{E}(x,y,z,t) = \vec{i}\cdot E(x,t) + \vec{j}\cdot E(y,t) + \vec{k}\cdot E(z,t)$$

where $\vec{i}$, $\vec{j}$ and $\vec{k}$ are *unit vectors*, vectors of magnitude one and direction parallel to the x, y and z coordinate axes respectively. $E(x,t)$ is the scalar (magnitude, only, without direction) component in the x direction and is usually referred to as E_x, and analogously for E_y and E_z. In this manner mathematics can be performed on the magnitudes in the usual fashion.

Derivatives of vector quantities may be vectors, also or they may be scalar, depending on the definition. There are two special derivatives involved in Maxwell's Equations. The first, symbolized $\nabla \cdot$ *[···]* and called *the divergence of [···]*, is as follows.

(DN7-4) $$\nabla \cdot \vec{E} \equiv \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$ [a scalar quantity not a vector]

This *divergence* measures the scalar rate of change of the vector quantity outward in all three dimensions.

The second special derivative, which is symbolized $\nabla \times$*[···]* and called *the curl of [···]* is a vector quantity as follows.

(DN7-5) $$\nabla \times \vec{E} \equiv \vec{i}\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] + \vec{j}\left[\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right] + \vec{k}\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$$

One can visualize a sheet of paper *curled* into a roll. One term of the curl expression above is a measure of the tightness with which the paper sheet is rolled. That is, if the axis around which the paper is curled is the x direction then the first term of equation DN7-5 is the tightness of the curling of the paper (in the y and z directions, the curl of the paper being a circle in the y - z plane). Of course it is more difficult to visualize something curled in all three directions at the same time, which is what the overall curl expression describes.

These are derivatives in space. The divergence describes the change in E along each dimension. The curl describes the change in E in each dimension relative to the other two dimensions. (Curl reflects and embodies the right angle behavior of magnetic effect directions as described at the beginning of section 14.)

In Maxwell's Equations the electric and magnetic fields are described in terms of field strength, which is the factor related to the force that the field exerts on a charge. For electric field the field strength is symbolized E and for magnetic field H. The magnitude of the force exerted on charge Q_2 by the electric field of charge Q_1 is

(DN7-6) $$F = Q_2 \cdot E$$

and, since from Coulomb's Law

(DN7-7) $$F = \frac{Q_1 \cdot Q_2}{4\pi \cdot \varepsilon \cdot R^2}$$

then the field is

(DN7-8) $$E = \frac{F}{Q_2} = \frac{Q_1}{4\pi \cdot \varepsilon \cdot R^2}$$

Likewise, for magnetic field

(DN7-9) $F = I_2 \cdot H$

and, from Ampere's Law

(DN7-9) $$F = \frac{\mu}{2\pi} \cdot \frac{I_1 \cdot I_2}{R} \cdot L$$

so that

(DN7-10) $$H = \frac{F}{I_2} = \frac{\mu \cdot I_1}{2\pi \cdot R} \cdot L$$

Combining all of the foregoing terminologies, Maxwell's Equations are as follows (the four verbal statements of the laws now converted to mathematical expressions).

I - The curl of the magnetic field strength is equal to the current density plus the time rate of change of the electric field strength,

(DN7-11) $$\nabla \times \overline{H} = \iota_c + \varepsilon \cdot \frac{\partial \vec{E}}{\partial t} \qquad [\iota_c \equiv \text{current density}]$$

which states that current flow or changing electric field produces magnetic field.

II - The curl of the electric field strength is equal to minus the time rate of change of the magnetic field strength,

(DN7-12) $$\nabla \times \overline{E} = -\mu \cdot \frac{\partial \vec{H}}{\partial t}$$

which states that changing magnetic field produces electric field.

III - Electric field diverges from charge or closes on itself (if $\rho = 0$).

(DN7-13) $$\nabla \cdot [\varepsilon \cdot \vec{E}] = \rho \qquad [\rho \equiv \text{charge density}]$$

IV - Magnetic field closes upon itself.

(DN7-14) $$\nabla \cdot [\mu \cdot \vec{H}] = 0.$$

These equations describe all electromagnetic phenomena. In the appropriate configuration they describe a time varying wave traveling in space with a velocity

(DN7-15) $c = {}^1/_{\sqrt{\mu \cdot \varepsilon}}$

which is electromagnetic radiation.